ISSN 1041-9489

SAFETY BRIEF

November, 2003 Volume 24, No. 3

Triodyne Inc.
Consulting Engineers & Scientists – Safety Philosophy & Technology
666 Dundee Road, Suite 103, Northbrook, IL 60062-2702
(847) 677-4730 FAX: (847) 647-2047
e-mail: infoserv@triodyne.com
www.triodyne.com

FLOOR RELIABILITY WITH RESPECT TO "SLIP AND FALL"

Ralph Lipsey Barnett* and Peter Joseph Poczynok**

ABSTRACT

For a given community of walkers and a specific type of ambulation, force-plate studies have established the required level of horizontal resistance for stable locomotion. This stochastic floor loading is resisted by friction forces which must be great enough to prevent slipping. A statistical characterization of frictional resistance has recently been developed using extreme value statistics. Reliability theory provides a method for combining the floor loading and friction resistance which, for the first time, enables one to determine in a rational manner the probability of slipping. This paper presents a formula describing the "slip and fall" reliability of a floor/footwear couple.

I. INTRODUCTION

During ambulation, how many walkers slip? Conventional "slip and fall" theory does not address this question; it merely establishes a go/no-go criterion that indicates whether or not a given floor is satisfactory. Specifically, the theory states that no slip, and hence no fall, will occur whenever the average coefficient of friction $\bar{\mu}$ between a floor and some standard footwear material, say leather, is greater than a critical friction coefficient μ_c, i.e.,

$$\bar{\mu} > \mu_c \text{...no slip}$$

The critical friction criterion μ_c is not selected by some rational protocol; it is often established by legislative fiat or consensus.

The concept of a critical friction criterion is not helpful when trying to predict the probability that an individual walker will slip during a particular ambulating scenario. This undertaking is the focus of the present paper which integrates ideas from three different disciplines: force-plate studies, friction characterization studies, and reliability theory.

A. Force-Plate

Gait laboratories measure the force applied to a surface by various communities of users during specific types of ambulation such as straight walking or turning. They use an instrumented walking surface called a force-plate that records the time history of both the horizontal force component $H(t)$ and the corresponding vertical force component $V(t)$ impressed on the surface by walking candidates. Throughout a typical step, the horizontal applied floor loading $H(t)$ must be resisted if no slip is to occur. This resistance is developed by the normal surface loading $V(t)$ acting in conjunction with the coefficient of friction μ between the surface and the walker's footwear. At any time t the non-slip criterion may be written as:

No Charge

* Professor, Mechanical and Aerospace Engineering, Illinois Institute of Technology, Chicago, IL and Chairman, Triodyne Inc., Northbrook, IL.
** Senior Mechanical Engineer, Triodyne Inc., Northbrook, IL.

$H(t) < \mu V(t)$... non-slip

To survive the entire step without slipping,

$$(H/V)_{max} < \mu \text{ ...non-slip} \qquad \text{Eq. 1}$$

where the largest ratio $(H/V)_{max}$ is taken in the one-step interval.

In a typical force-plate study by Harper, Warlow, and Clark (1961), maximum values of *H/V* were recorded for a level surface for men and women during straight walks and turns. Their force-plate measurements of $(H/V)_{max}$, which are summarized in Table I, represent 87 sets of data for men and 37 sets for women. They characterized the data using a normal distribution; the mean, standard deviation, and 99.9999 percentile are tabulated in Table I for each of their six test programs. Using statistical inference, Harper et al. estimated the $(H/V)_{max}$ at the 99.9999 percentile level for straight walking males, $(H/V)_{max} = 0.36$. This implies that only one in a million men will exceed this value.

Let the required frictional resistance or applied floor loading $(H/V)_{max}$ be designated as μ_a. If this applied loading is represented by a normal or Gaussian distribution, the probability density function $\tilde{f}$ may be written,

$$\tilde{f}(\mu_a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2} \qquad \text{Eq. 2}$$

where $\bar{\mu} = \overline{(H/V)_{max}}$ is the mean value of the $(H/V)_{max}$ distribution, σ is its standard deviation, and μ_a takes on values from minus to plus infinity. As usual, the probability that the applied floor friction does not exceed μ_a, $Pr\{(H/V)_{max} \le \mu_a\}$, is given by the cumulative distribution function P, i.e.,

$$\mathrm{P}(\mu_a) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\mu_a} e^{-\frac{1}{2}\left(\frac{\mu - \bar{\mu}}{\sigma}\right)} d\mu$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\mu_a - \bar{\mu})/\sigma} e^{-t^2/2} dt = \Phi\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right) \equiv \Phi(z) \qquad \text{Eq. 3b}$$

where Φ is the standardized normal distribution which is a tabulated function that appears in almost every book on statistics. It should be noted that

$$\Phi(-z) = 1 - \Phi(z) \qquad \text{Eq. 4}$$

B. Slip Resistance

If the coefficient of friction is measured throughout a walking surface, the resulting values may be presented as a "bell shaped" curve which characterizes the floor/footwear couple. To execute an n-step perambulation across the surface without slipping requires that a walker survive the step with the lowest friction. This observation has led to the development of a new theory of "slip and fall" based on extreme value statistics (Barnett, 2002). This theory provides that the "bell shaped" curve of friction coefficients must be of the Weibull form and that the probability that a random friction coefficient M will not exceed μ_r, $\mathrm{P}_r\{M \le \mu_r\}$, is expressed by F:

$$F(\mu_r) = 1 - e^{-n\left(\frac{\mu_r - \mu_z}{\mu_o}\right)^m} \quad ...\mu_r \ge \mu_z \qquad \text{Eq. 5}$$

$$= 0 \qquad ...\mu_r \le \mu_z$$

where μ_r is the resisting coefficient of friction for a particular floor/footwear couple; n is the number of steps taken during a given walk, and μ_o, μ_z and m are Weibull parameters obtained from the data represented by the "bell shaped"

Table I - Maximum H/V (after Harper, Warlow, and Clark, 1961)

	Straight Walking		Turning			
			Left Foot		Right Foot	
Statistical Properties	Men	Women	Men	Women	Men	Women
Mean	0.17	0.16	0.19	0.17	0.22	0.19
Standard Deviation	0.04	0.03	0.04	0.02	0.04	0.02
99.9999 Percentile	0.36	—	0.40	—	0.36	—

probability density function. It should be noted that μ_z is the zero probability friction coefficient; for applied loads at or below this value there is no risk of slipping.

Anecdotal evidence reflecting decades of widespread use of one-foot square asphalt floor tiles has shown them to provide outstanding low-slip floor surfaces. One would expect the associated friction resistance of these floors to be highly successful in resisting the applied floor loadings developed by Harper *et al.*

Following the test protocol specified by ASTM F609-79 (ASTM, 1989b), 400 coefficients of friction were obtained between 100 new one-foot square asphalt tiles and three 0.5 inch (1.27-cm) diameter leather specimens under dry conditions. The sample data are presented as a histogram in Figure 1 and as a cumulative distribution function $F(\mu)$ in Figure 2. A continuous Weibull probability density curve $f(\mu)$ was fitted to the data in the histogram using the parameters $\mu_z = 0.31$, $\mu_o = 0.40$ and $m = 4.75$.

II. THEORY OF FLOOR RELIABILITY

The probability that a walker will not slip, and hence not fall, is called reliability and it will be designated by R. When the applied floor loading μ_a and the friction resistance of a floor/footwear couple μ_r are both stochastic, the floor reliability R may be determined by well established techniques developed in reliability theory. These techniques are all predicated on the observation that failure (slip) will not occur if resistance (strength) is greater than loading (stress); for non-slip this implies that $\mu_r > \mu_a$. Reliability theory almost always extends the no-failure criterion to include the equality of strength and load. Since this assumption has no influence on our slip predictions, consistency motivates the following definition:

$$(\mu_r - \mu_a) \geq 0 \text{...no slip criterion...} \qquad \text{Eq. 6}$$

The most general probability density function $\tilde{f}(\mu_a)$ is shown in Fig. 3a for applied floor loading μ_a where μ_a ranges from minus to plus infinity. A corresponding probability density function $f(\mu_r)$ is illustrated in Fig. 3b for floor resistance μ_r which in the most general case will also span from minus to plus infinity. In a given step, the probability that a walker will apply a specific floor loading μ_a is given by $\tilde{f}(\mu_a)d\mu_a$ as indicated by the hatched area in Fig. 3a. Observe in Fig. 3b that the shaded area represents all of the resisting friction coefficients that are equal to or greater than μ_a. This area is the probability that a walker will encounter frictional resistance equal to or greater than μ_a:

$$\Pr\{\mu_r \geq \mu_a\} = \int_{\mu_a}^{\infty} f(\mu_r)d\mu_r \qquad \text{Eq. 7}$$

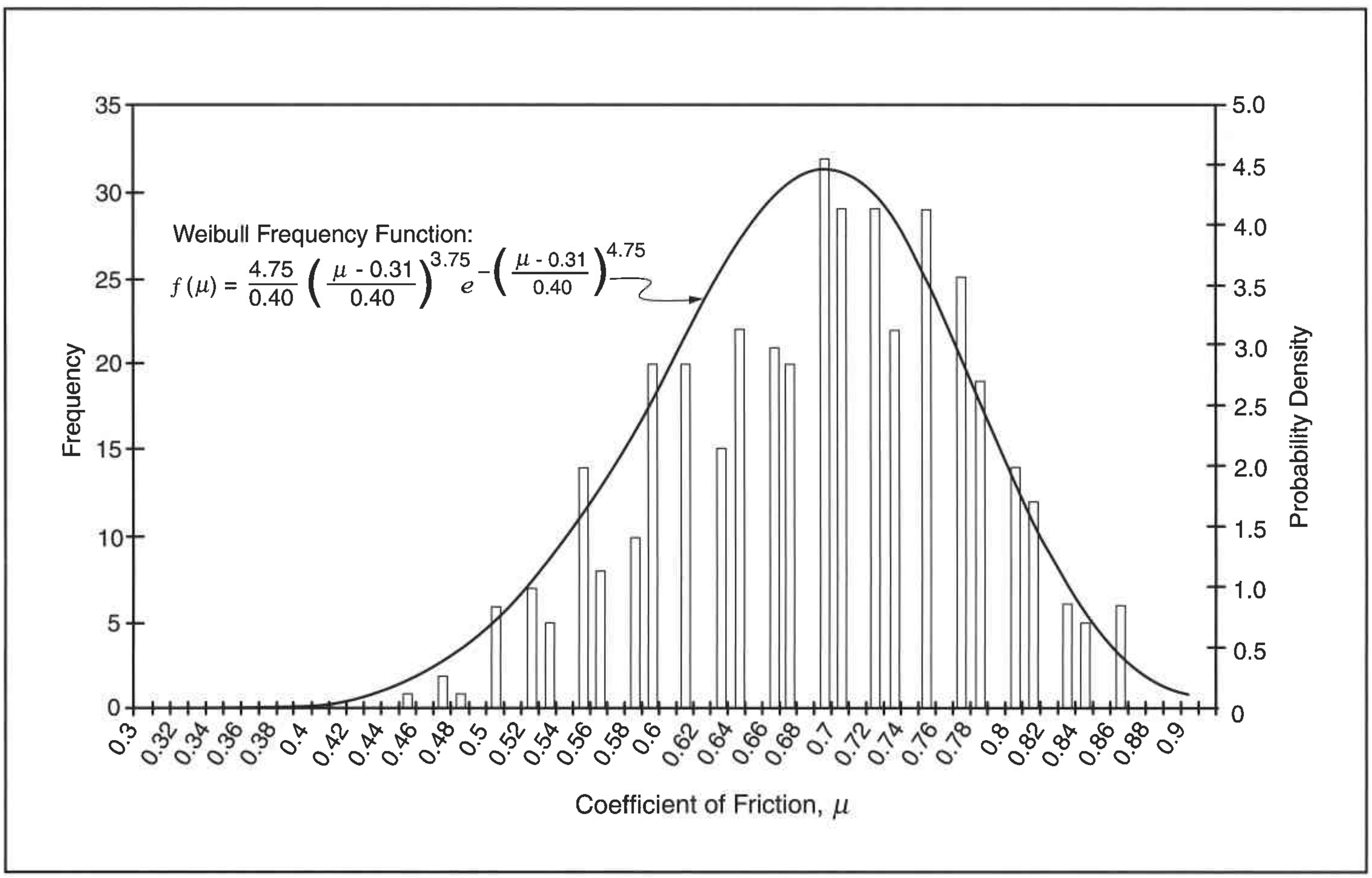

Figure 1 - Histogram: Coefficients of Friction

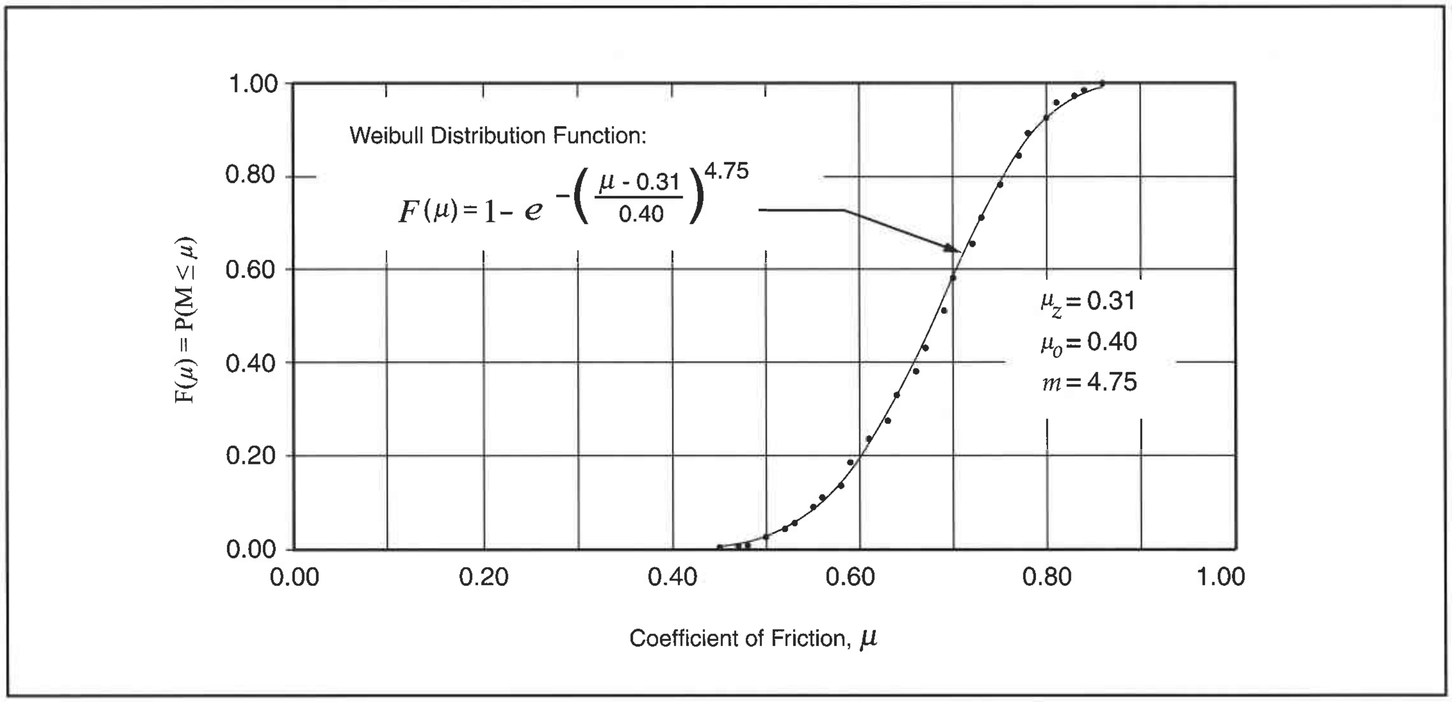

Figure 2 - Cumulative Distribution Function: Coefficients of Friction

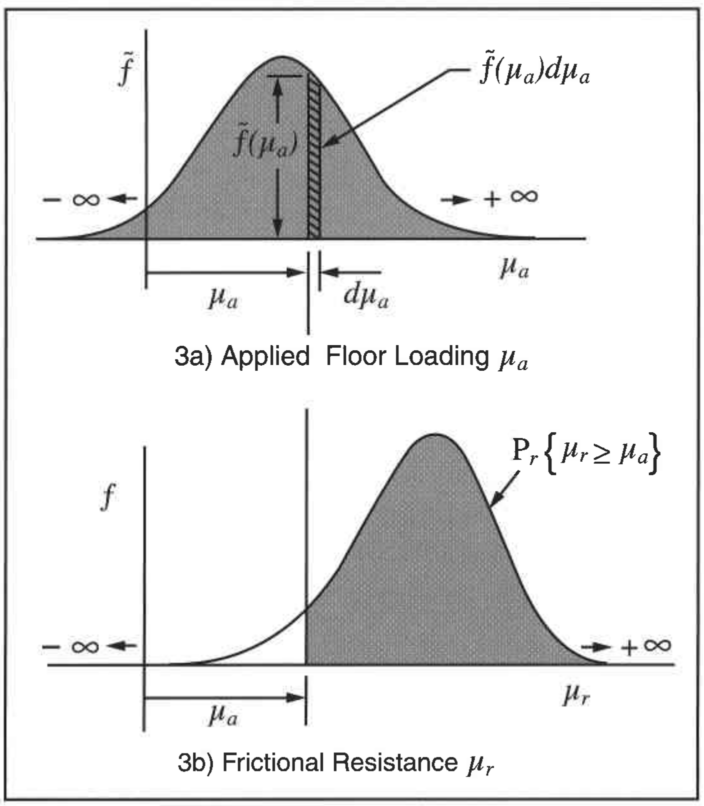

Figure 3 - Probability Density Functions,
$\tilde{f}(\mu_a)$ and $f(\mu_r)$

The reliability dR is the probability of not slipping, i.e., the probability of simultaneously achieving the resistance $\mu_r \geq \mu_a$ whenever the loading μ_a is impressed on the floor. Because loading and resistance are independent phenomena, the reliability dR is the product of the loading probability $\tilde{f}(\mu_a)d\mu_a$ and $\Pr\{\mu_r \geq \mu_a\}$; thus,

$$dR = \tilde{f}(\mu_a)d\mu_a \int_{\mu_a}^{\infty} f(\mu_r)d\mu_r \qquad \text{Eq. 8}$$

The total reliability R is the sum of all the reliabilities dR associated with every possible floor loading μ_a from minus to plus infinity, hence,

$$R = \int_{-\infty}^{\infty} \tilde{f}(\mu_a)\left[\int_{\mu_a}^{\infty} f(\mu_r)d\mu_r\right]d\mu_a \qquad \text{Eq. 9}$$

This well known reliability formula is discussed extensively by Kececioglu and Cormier (1964) together with various other methods for solving the reliability problem when applied loading and resistance distributions are non-normal. All of these methods ultimately appeal to numerical evaluation. The only known exact analytical procedures for solving the reliability of stress/strength problems require that $\tilde{f}$ and f are both normal or both log-normal distributions. The availability of exact solutions is comforting when evaluating different numerical protocols before they are applied to real world problems. For this reason Appendix I demonstrates the classical reliability calculations when both $\tilde{f}$ and f are normal distributions.

III. FLOOR RELIABILITY: NORMAL AND WEIBULL DISTRIBUTIONS

Rewrite the general form for reliability given by Eq. 9 where $\tilde{f}(\mu_a)$ is represented by the normal distribution given by Eq. 2 and $f(\mu_r)$ reflects the probability density function associated with the Weibull distribution described by Eq. 5;

$$R = \int_{-\infty}^{\infty} \tilde{f}(\mu_a)\left[\int_{\mu_a}^{\infty} f(\mu_r)d\mu_r\right]d\mu_a \qquad \text{Eq. 9}$$

Because the total area beneath $f(\mu_r)$ is unity,

$$\int_{\mu_a}^{\infty} f(\mu_r)d\mu_r = 1 - \int_{-\infty}^{\mu_a} f(\mu_r)d\mu_r \qquad \text{Eq. 10}$$

Substituting this result into Eq. 9, R becomes

$$R = \int_{-\infty}^{\infty} \tilde{f}(\mu_a)\left[1 - \int_{-\infty}^{\mu_a} f(\mu_r)d\mu_r\right]d\mu_a$$

where, by definition,

$$\int_{-\infty}^{\mu_a} f(\mu_r)d\mu_r \equiv F(\mu_a)\ldots \text{ Weibull}$$

Hence,

$$R = \int_{-\infty}^{\infty} \tilde{f}(\mu_a)[1 - F(\mu_a)]d\mu_a \qquad \text{Eq. 11}$$

where $$F(\mu_a) = 1 - e^{-n\left(\frac{\mu_a - \mu_z}{\mu_o}\right)^m} \ldots \mu_a \geq \mu_z$$

$$= 0 \qquad \ldots \mu_a \leq \mu_z$$

Let R_1 represent the reliability in the applied floor loading range $-\infty \leq \mu_a \leq \mu_z$; let R_2 reflect the reliability in the range $\mu_z \leq \mu_a \leq \infty$ where μ_z is the zero probability friction resistance. Then,

for $-\infty \leq \mu_a \leq \mu_z$:

$$R_1 = \int_{-\infty}^{\mu_z} \tilde{f}(\mu_a)[1 - 0]d\mu_a$$

$$= \int_{-\infty}^{\mu_z} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2} d\mu_a$$

$$= P(\mu_z) = \Phi\left(\frac{\mu_z - \bar{\mu}}{\sigma}\right) \qquad \text{Eq. 12}$$

for $\mu_z \leq \mu_a \leq \infty$:

$$R_2 = \int_{\mu_z}^{\infty} \tilde{f}(\mu_a)\left\{1 - \left[1 - e^{-n\left(\frac{\mu_a - \mu_z}{\mu_o}\right)^m}\right]\right\}d\mu_a$$

$$= \int_{\mu_z}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}_a}{\sigma}\right)^2} e^{-n\left(\frac{\mu_a - \mu_z}{\mu_o}\right)^m} d\mu_a$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu_z}^{\infty} e^{-\left[\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2 + n\left(\frac{\mu_a - \mu_z}{\mu_o}\right)^m\right]} d\mu_a \qquad \text{Eq. 13}$$

The total reliability is

$$R = R_1 + R_2 \qquad \text{Eq. 14}$$

R_1 is obtained by consulting a normal distribution table. R_2 must be evaluated by numerical integration.

Example 1:

Using data from Table I and Figs. 1 and 2, the reliability of an asphalt tile floor can be calculated for straight walking scenarios involving men wearing leather footwear. Thus,

Applied Loading (Gaussian)	Resistance (Weibull)
$\bar{\mu} = 0.17$	$m = 4.75$
$\sigma = 0.04$	$\mu_z = 0.31$
$n = 1, 10, 100, 1000, 10{,}000$	$\mu_o = 0.40$

for $\mu_a \leq \mu_z$:

$$R_1 = \Phi\left(\frac{\mu_z - \bar{\mu}}{\sigma}\right) = \Phi\left(\frac{0.31 - 0.17}{0.04}\right)$$

$$= \Phi(3.5) = 0.99976\ 73709$$

where Φ was taken from the Handbook of Mathematical Functions (1964). Note that floor reliability under applied loads below the zero probability friction does not depend on the length of a walker's excursion.

for $\mu_a \geq \mu_z$:

$$R_2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu_z}^{\infty} e^{-\left[\frac{1}{2}\left(\frac{\mu_a-\bar{\mu}}{\sigma}\right)^2 + n\left(\frac{\mu_a-\mu_z}{\mu_o}\right)^m\right]} d\mu_a$$

$$= \frac{1}{0.04\sqrt{2\pi}} \int_{0.31}^{1} e^{-\left[\frac{1}{2}\left(\frac{\mu_a-0.17}{0.04}\right)^2 + n\left(\frac{\mu_a-0.31}{0.40}\right)^{4.75}\right]} d\mu_a$$

where unity replaces the upper bound which will not be reached by the coefficient of friction. R_2 was numerically integrated using Simpson's four interval rule with a computer spreadsheet:

$$\int_{x_o}^{x_4} f(x)dx = \frac{2h}{45}[7f(x_o) + 32f(x_1) + 12f(x_2)$$

$$+32f(x_3) + 7f(x_4)] - \frac{8h^7}{945} f^{(6)}(\xi) \ldots \xi \; in(x_o, x_4)$$

where h, which is the length of each of the four equal intervals, was taken as h = 0.0025. This integration rule is developed in Applied Numerical Methods by Carnahan, Luther, and Wilkes (1969). The computations are tabulated in Table II for the various values of n.

Table II clearly demonstrates that the asphalt tiles provide an almost slip-free walking surface. Taking the length of a man's step as 2.7 ft., the last entry in Table II implies that less than 3 slips will occur during 5 million miles of walking.

Example 2

The superior resistance of the asphalt tile floor is due in large measure to its high zero probability friction, $\mu_z = 0.31$. Reconsider Example 1 where $\mu_z = 0$. Thus,

for $\mu_a \leq \mu_z = 0$:

$$R_1 = \Phi\left(\frac{\mu_z - \bar{\mu}}{\sigma}\right) = \Phi\left(\frac{-0.17}{0.04}\right)$$

$$= \Phi(-4.25) = 1 - \Phi(4.25)$$

$$= 0.00001\ 06885$$

for $\mu_a \geq \mu_z = 0$:

$$R_2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu_z}^{\infty} e^{-\left[\frac{1}{2}\left(\frac{\mu_a-\bar{\mu}}{\sigma}\right)^2 + n\left(\frac{\mu_a-\mu_z}{\mu_o}\right)^m\right]} d\mu_a$$

$$= \frac{1}{0.04\sqrt{2\pi}} \int_{0}^{1} e^{-\left[\frac{1}{2}\left(\frac{\mu_a-0.17}{0.04}\right)^2 + n\left(\frac{\mu_a}{0.40}\right)^{4.75}\right]} d\mu_a$$

R_2 was numerically integrated using the "built-in" program in a Hewlett Packard, Model HP-20S hand calculator. This software uses the Simpson's two interval rule:

$$\int_{x_o}^{x_2} f(x)dx = \frac{h}{3}[f(x_o) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi)$$

$$\ldots \xi \; in(x_o, x_2)$$

Here, the equal intervals h must be even; h was taken as h = 0.01. The R_2 calculations are tabulated in Table III.

Table II - Floor Reliability: Asphalt Tile / Leather Footwear / Men / Straight Walking

Number of Steps n	$\mu_a \leq \mu_z$ R_1	$\mu_a \geq \mu_z$ R_2	Reliability $R = R_1 + R_2$	Probability of Slipping $1 - R$	Slips Per Million Walkers
1	0.99976 73709	$2.326\ 287\ 885 \times 10^{-4}$	0.999 999 999	3.11×10^{-10}	zero
10	0.99976 73709	$2.326\ 260\ 268 \times 10^{-4}$	0.999 999 997	3.07×10^{-9}	zero
100	0.99976 73709	$2.325\ 985\ 555 \times 10^{-4}$	0.999 999 969	3.05×10^{-8}	zero
1000	0.99976 73709	$2.323\ 361\ 925 \times 10^{-4}$	0.999 999 707	2.93×10^{-7}	0.293
10,000	0.99976 73709	$2.303\ 100\ 954 \times 10^{-4}$	0.999 997 681	2.32×10^{-6}	2.32

The imaginary floor represented in Table III is clearly a menace. The zero probability friction plays an important role in providing slip resistance. If this floor were used in an airport where walkers normally exceed 1000 steps, almost everyone would slip, although they would not necessarily fall.

DISCUSSION:

1. The two major research components of current "slip and fall" theory are force-plate studies and tribometry studies. These two areas are loosely combined under the mantle of an irrational inequality comparing the average coefficient of friction of a floor/footwear couple to a critical friction criterion that may be characterized as a friction fiction.

2. The present paper provides a linchpin for a rational "slip and fall" theory that combines several disparate disciplines. An appeal is made to reliability theory to compare floor loading and floor resistance. Floor loading is described by force-plate studies and by the characterization of floor duty cycles by Barnett, Poczynok & Glowiak (2002). Floor resistance involves both the study of tribometers and the interpretation of data using a formulation of slip resistance based on extreme value statistics by Barnett (2002).

3. The probability that walkers will not slip and fall, floor reliability, is given by the double integral, Eq. 9. This formula must always be evaluated numerically.

4. The reliability of a ubiquitous asphalt tile floor under straight walking scenarios was calculated for various length walks. For all practical purposes the reliability was 100%. Less than three slips occurred during five million miles of walking. This result was anticipated based on anecdotal experience.

5. The same walking scenario evaluated for the asphalt tile floor was applied to an imaginary floor with an average coefficient of friction of 0.366 and a standard deviation of 0.088; the coefficient of variation is 24%. The corresponding Weibull parameters are $\mu_z = 0,\ \mu_o = 0.40$, and $m = 4.75$. By conventional standards this floor is dreadful. Our calculations showed that walks of 10 steps caused 20% of the walkers to slip; 1000 step walks gave rise to 97% slips.

6. If the safety of floors is judged at a level of a few slips per million walkers, the accuracy implications are quite far-reaching. Are 87 sets of force-plate data sufficient for predicting the behavior of a million men? Do 400 coefficients of friction characterize a floor for $n \times 10^6$ steps? How does one reflect the duty cycle of a floor with great precision? Fortunately, numerical integration can easily provide seven significant figures with accuracy and economy.

7. The proposed protocol has the advantage of not requiring "slip and fall" studies which directly record a slipping incident. Slipping is a dangerous activity that may be difficult to identify when not accompanied by falling.

8. If walkers do not slip, they will not fall. The converse is untrue; walkers that slip do not necessarily fall.

9. Judgment is required to decide what level of floor reliability constitutes an acceptable floor for a given community of users.

REFERENCES

Abramowitz, M. and I. A. Stegun (eds.), *Handbook of Mathematical Functions.* National Bureau of Standards Applied Mathematics Series No. 55. Washington, DC., U.S. Government Printing Office, 1964.

American Society for Testing and Materials (ASTM). (1989b). *Standard test tethod for static slip resistance for footwear, sole, heel, or related materials by horizontal pull slipmeter (HPS).* (Standard No. ASTM F609-79 [Reapproved 1989]). Philadelphia, Pennsylvania: Author.

Barnett, Ralph L., "Slip and Fall" Theory - Extreme Order Statistics," *International Journal of Occupational Safety and Ergonomics*, Vol. 8 No. 2 (2002): 135-160.

Barnett, Ralph L., Suzanne A. Glowiak, and Peter J. Poczynok, "Stochastic Theory of Human Slipping," *Safety Engineering and Risk Analysis*, SERA - Vol. 12, Las Vegas, NV, American Society of Mechanical Engineers, November 2002.

Carnahan, Brice, H.A. Luther and James O. Wilkes, *Applied Numerical Methods.* New York, John Wiley & Sons, Inc., 1969.

Table III - Floor Reliability with $\mu_z = 0$ and Example 1 Data

Number of Steps n	$\mu_a \le 0$ R_1	$\mu_a \ge 0$ R_2	Reliability $R = R_1 + R_2$	Probability of Slipping $1 - R$	Slips Per Million Walkers
1	0.00001 06885	0.97451 5049	0.97452 574	0.025 474	25,474
10	0.00001 06885	0.79514 3200	0.79515 389	0.204 846	204,846
100	0.00001 06885	0.27782 3244	0.27783 393	0.722 166	722,166
1000	0.00001 06885	0.02947 7417	0.02948 810	0.970 512	970,512

Harper, F.C., W.J. Warlow and B.L. Clarke (1961). The forces applied to the floor by the foot in walking: I. Walking on a level surface. *National Building Studies: Research Paper 32.* London: Department of Scientific and Industrial Research Building Research Station.

Hoel, Paul G., *Introduction to Mathematical Statistics.* New York, John Wiley & Sons, Inc., 1971, p. 133.

Kececioglu, Dimitri and David Cormier, “Designing a Specified Reliability Directly into a Component,” in *Proceedings of the Third Annual Aerospace Reliability and Maintainability Conference, June 29 - July 1, 1964*, New York, Society of Automotive Engineers, 1964, pp. 546-565.

APPENDIX I

Reliability: Normally Distributed Applied Friction Loading and Resistance

The reliability R of a floor/footwear couple is the probability that its resistance μ_r is equal to or greater than the applied friction loading μ_a (Eq. 6);

$$(\mu_r - \mu_a) \geq 0 \ldots \text{ no slip}$$

If $f(\mu_r)$ is normally distributed with mean $\bar{\mu}_r$ and standard deviation σ_r and if $\tilde{f}(\mu_a)$ is also distributed normally with mean $\bar{\mu}$ and standard deviation σ , the difference of the two variates $\xi \equiv (\mu_r - \mu_a)$ is also a normal distribution with mean $\bar{\xi}$ and standard deviation σ_ξ (Hoel, 1971),

where

$$\bar{\xi} = \bar{\mu}_r - \bar{\mu} \qquad \text{Eq. 15}$$

and

$$\sigma_\xi = \sqrt{\sigma_r^2 + \sigma^2} \qquad \text{Eq. 16}$$

Figure 4 shows a normal probability density function for the difference quantity ξ. The area of the shaded portion of this figure represents the reliability of the associated floor/footwear couple, i.e.,

$$R = \int_o^\infty f(\xi)d\xi = 1 - \frac{1}{\sigma_\xi\sqrt{2\pi}}\int_{-\infty}^{o} e^{-\frac{1}{2}\left(\frac{\xi-\bar{\xi}}{\sigma_\xi}\right)^2} d_\xi$$

$$= 1 - \Phi\left(\frac{0-\bar{\xi}}{\sigma_\xi}\right)$$

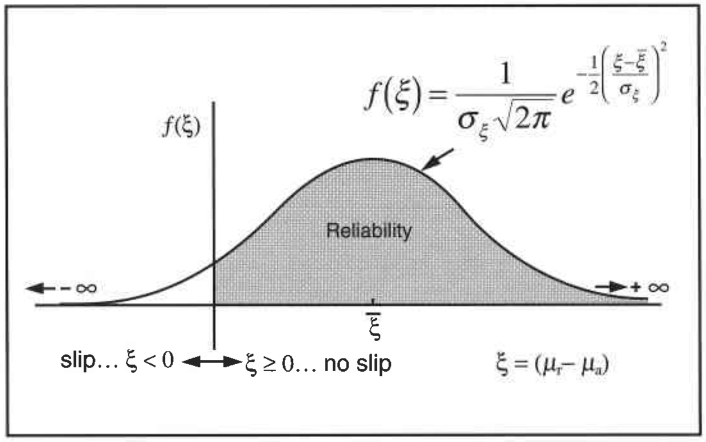

Figure 4 - Distribution of Difference Function $\xi = (\mu_r - \mu_a)$

$$= 1 - \Phi\left[\frac{-(\bar{\mu}_r - \bar{\mu})}{\sqrt{\sigma_r^2 + \sigma^2}}\right] \qquad \text{Eq.17}$$

Example 3

For purposes of this example, we will assume that the distribution of friction coefficients is normal (recall that it must be Weibull). Thus,

Applied Loading:	$\bar{\mu} = 0.17; \sigma = 0.04$
Resistance:	$\bar{\mu}_r = 0.50; \sigma_r = 0.111$

Then,

$$R = 1 - \Phi\left[\frac{-(0.50-0.17)}{\sqrt{(0.111)^2 + (0.04)^2}}\right] = 1 - \Phi(-2.8)$$

$$= 1 - [1 - \Phi(2.8)] = \Phi(2.8) = 0.99744 \;\; 48696$$

SAFETY BRIEF

November 2003 – Volume 24, No. 3

Editor: Paula L. Barnett

Illustrated and Produced by

Triodyne Graphic Communication