

# SAFETY BRIEF

## MECHANICAL ENGINEERING:

**Triodyne Inc.**  
*(Est. 1969)*

Officers  
 Ralph L. Barnett  
 Dolores Gildin  
 S. Carl Uzgriris, Ph.D.

## Mechanical Engineering

Ralph L. Barnett  
 Dennis B. Brickman  
 Michael A. Dilich  
 Christopher W. Ferrone  
 Suzanne A. Glowiak  
 John M. Goebelbecker  
 Audra E. Gray  
 Crispin Hales, Ph.D.  
 Dror Kopernik  
 Woodrow Nelson  
 Cheryl A. Patten, Ph.D.  
 Peter J. Poczynok  
 Audrone M. Stake, Ph.D.  
 William G. Switalski  
 George J. Trezek, Ph.D.  
 S. Carl Uzgriris, Ph.D.  
 James R. Wingfield, Ph.D.

Library Services  
 Marna S. Sanders  
 Betty Bellows  
 Donna Klick  
 John Kristelli  
 Florence Lasky  
 Donna Spencer

Information Products  
*Expert Transcript*  
*Center (ETC)*  
 Marna S. Sanders

Graphic Communications  
 Robert Kautny  
 Charles D'Eccliss

Training and Editorial Services  
 Paula L. Barnett

Vehicle Laboratory  
 Charles Sinkovits  
 Matthew J. Ulmenstine

Model Laboratory  
 2721 Alison Lane  
 Wilmette, IL 60091-2101  
 Bill Brown

Photographic Laboratory  
 7903 Beckwith Road  
 Morton Grove, IL 60053  
 Larry Good

Business Systems  
 Chris Ann Gonatas  
 Cheryl Black  
 Rita Curtis  
 Sandra Prieto  
 Sandie Christiansen  
 Jaimie Santiago

Facilities Management  
 Peter W. Warner

## SAFETY PRODUCTS:

**Triodyne Safety**  
**Systems, L.L.C.**

*(Est. 1998)*  
 5950 West Touhy Avenue  
 Niles, IL 60714-4610  
 (847) 677-4730  
 FAX: (847) 647-2047

Officers/Directors  
 Ralph L. Barnett  
 Paula L. Barnett  
 Joel I. Barnett

President  
 Peter J. Poczynok  
 Vice President of Operations  
 Peter W. Warner

Senior Science Advisor  
 Theodore Liber, Ph.D.

Mechanical Engineering  
 Ralph L. Barnett  
 Peter J. Poczynok  
 Aquatics Safety Consultant  
 Ronald M. Schroader

February, 2003


**Triodyne Inc.**

Consulting Engineers &amp; Scientists – Safety Philosophy &amp; Technology

5950 West Touhy Avenue Niles, IL 60714-4610

(847) 677-4730 FAX: (847) 647-2047

e-mail: infoserv@triodyne.com www.triodyne.com

Volume 22, No. 4

SAFETY RESEARCH:  
**Institute for Advanced**  
**Safety Studies**

*(Est. 1984)*  
 5950 West Touhy Avenue  
 Niles, IL 60714-4610  
 (847) 647-1101

Chairman  
 Ralph L. Barnett  
 Director of Operations  
 Paula L. Barnett  
 Information Services  
 Marna S. Sanders  
 Senior Science Advisor  
 Theodore Liber, Ph.D.

## MANUFACTURING:

**Alliance Tool &**  
**Manufacturing Inc.**

*(Est. 1945)*  
 91 East Wilcox Street  
 Maywood, IL 60153-2397  
 (773) 281-1712  
 (708) 345-5444  
 FAX: (708) 345-4004

Officers  
 S. Carl Uzgriris, Ph.D.  
 Ralph L. Barnett  
 General Manager  
 Ramesh Gandhi  
 Plant Manager  
 Bruno Stachon  
 Founders/Consultants  
 Joseph Gansacz  
 Albert Kanikula

## CONSTRUCTION:

**Triodyne-Wangler**  
**Construction Company Inc.**

*(Est. 1993)*  
 5950 West Touhy Avenue  
 Niles, IL 60714-4610  
 (847) 647-8866  
 FAX: (847) 647-0785

Officers/Directors/Managers  
 Joel I. Barnett  
 William A. Wangler  
 Joseph Wangler  
 Ralph L. Barnett

## BUILDING MAINTENANCE:

**Alliance Building**  
**Maintenance Corporation**

*(Est. 1999)*  
 5950 West Touhy Avenue  
 Niles, IL 60714-4610  
 (847) 647-1379  
 FAX: (847) 647-0785

Officers  
 David J. Smith  
 Joel I. Barnett  
 Ralph L. Barnett

## CONSULTANTS:

Richard M. Bilof, Ph.D.  
 Electromagnetic Compatibility

Richard Gullickson  
 Industrial Hygiene/Safety/  
 Chemistry

Beth A. Hamilton  
 Information Science  
 David W. Levinson, Ph.D.  
 Senior Metallurgical Advisor  
 Steven R. Schmid, Ph.D.  
 Food Processing Equipment

Diane Moshman  
 Chemical/Environmental  
 Engineering

Harry Smith  
 Electrical Engineering

Kim M. Mniszewski  
 Fire and Explosion

William A. Wangler  
 Construction  
 Joseph Wangler  
 Construction

## Stochastic Theory of Human Slipping

By Ralph L. Barnett\*, Peter J. Poczynok, P.E.\*\* and Suzanne A. Glowiak\*\*\*

### Abstract

The conventional approach to human slipping is essentially deterministic; it states that no slipping will occur when the average friction coefficient is greater than some critical friction criterion. Under this condition, pedestrians will not slip when they encounter the average friction coefficient. On the other hand, to successfully negotiate a walk of  $n$ -steps they must not slip when they encounter the smallest of the  $n$  friction coefficients. Consequently, a new slip theory has been formulated as a problem in extreme value statistics. An elegant relationship is obtained among the probability of slipping, the critical friction criterion, the number of steps taken by the walker, and the central measure, scatter, and asymmetry of the distribution of friction coefficients. The new theory reveals the structure of human slipping in a startling way that introduces completely new concepts: the go/no go nature of classical slip predictions is replaced by a probability of slipping; low friction floor/footwear couples may lead to fewer slips than high friction ones; slipping can occur in any case where conventional theory predicts "no slip"; and the number of slips depends on the distance traveled by a pedestrian. Finally, this paper develops the idea that the slipperiness of a real floor must be evaluated for a duty-cycle. Duty-cycles can be represented as frequency histograms when a floor is homogeneous and isotropic.

### I. Introduction

The traditional approach to "slip and fall" studies begins by measuring the coefficient of friction of a floor/footwear couple using one of over three dozen tribometry machines [Refs. 1 - 11]. The average of the friction coefficients,  $\bar{\mu}$ , characterizes the couple. This average is compared to a critical friction coefficient,  $\mu_c$ , to provide a non-slip criterion, i.e.,

$$\bar{\mu} > \mu_c \dots\dots\dots \text{no slip} \quad \text{Eq. 1}$$

As discussed extensively by Barnett [Ref. 12],  $\mu_c$  is always chosen to preclude the onset of slipping. If walkers do not slip, they will not fall. However, the converse is not true; walkers that slip do not necessarily fall. To simplify our discussions in this paper, it will be assumed that slipping occurs whenever a walker steps on a surface where  $\mu \leq \mu_c$ .

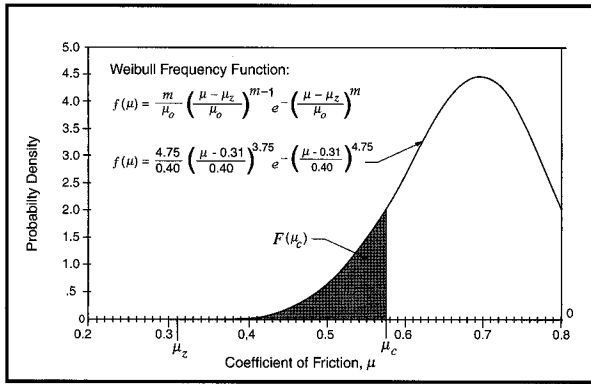
During a walk of  $n$ -steps, the smallest friction coefficient encountered must exceed  $\mu_c$  if slipping is to be avoided. To explore this notion an entire floor may be sampled to obtain the distribution of friction coefficients. These may be presented as a bell-shaped curve (probability density curve) or as the related cumulative distribution curve. A typical set of curves are shown in Fig. 1 which represent 400 static coefficients of friction obtained using a Horizontal Pull Slipmeter with three 0.5 (1.27 cm) inch diameter leather inserts from 100 new one foot square (30.5 cm x 30.5 cm) asphalt floor tiles. The testing protocol followed the specifications of ASTM F609-89 (1989b), [Ref. 13]. The curves have been fit with a

\* Professor, Mechanical and Aerospace Engineering, Illinois Institute of Technology, Chicago, and Chairman, Triodyne Inc., Niles, IL.

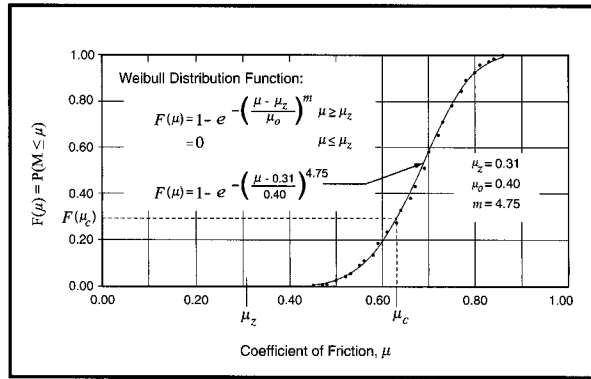
\*\* Senior Mechanical Engineer, Triodyne Inc., Niles, IL.

\*\*\* Senior Mechanical Engineer, Triodyne Inc., Niles, IL.

No Charge



a) Probability Density Curve



b) Cumulative Distribution Curve

Figure 1 - Distribution of Friction Coefficients: Asphalt Tile / Leather Couple

Weibull Distribution Function where  $\mu_z$  is the zero probability friction parameter and  $\mu_c$  is the critical friction coefficient.

It may be observed in Fig. 1a that the bell-shaped curve is not symmetrical. This implies that the highest point on the curve, the mode, will not represent the mean or average coefficient of friction. Because the mean  $\bar{\mu}$  does not represent the most probable value of  $\mu$ , it has no physical significance, only a mathematical one.

## II. Stochastic Slip Theory

Three concepts are involved in determining the probability that a pedestrian will slip during a preset walk. The first requires the characterization of friction properties for the floor/footwear couple. The second entails the development of a slip criterion, and the third involves a search for the “weakest link” or smallest friction coefficient encountered during the walk.

Friction measurements for a given floor/footwear couple are organized to establish the cumulative distribution function for the population of friction coefficients,  $F(\mu)$ . As usual, the distribution function describes the probability that a random friction coefficient  $\mu_r$  is less than or equal to  $\mu$ , i.e.,  $F(\mu) = P(\mu_r \leq \mu)$ . In the previous section,  $F(\mu)$  was taken as

a three parameter Weibull Distribution Function (see Fig. 1-b) to fit the measured data. The three Weibull parameters may be estimated by the method of moments [Ref. 14]. As a consequence, they are characterized by the central measure, the scatter, and the asymmetry of the probability density curve (see Fig. 1-a). Recall that classic slip theory uses only the central measure, the average friction  $\bar{\mu}$ .

We shall accept the notion that slip will occur whenever the friction coefficient  $\mu$  is less than or equal to the critical friction coefficient  $\mu_c$ :

$$\mu \leq \mu_c \dots \text{slip criterion} \quad \text{Eq. 2}$$

Consequently,  $F(\mu_c)$  is the probability that a random friction coefficient  $\mu_r$  is less than or equal to  $\mu_c$ , [ $F(\mu_c) = P(\mu_r \leq \mu_c)$ ]. Thus,  $F(\mu_c)$  is the probability of slipping when a single step is executed by a pedestrian.

If a pedestrian undertakes a walk involving  $n$  steps, his ambulation is successful if he does not slip during any one of the steps. Let  $F_w$  be the probability that the pedestrian will slip during the  $n$ -step walk. The probability that he will not slip is then  $[1 - F_w]$ . It may also be stated that  $[1 - F_w]$  is the probability that the pedestrian will sequentially survive all  $n$  steps. For any given step, the probability of slipping is represented by  $F(\mu_c)$ ; the probability of not slipping or surviving is simply  $[1 - F(\mu_c)]$ . We note that the condition prevailing at each step is independent of any other step. Invoking the Multiplication Rule for independent events [Ref. 15], the probability of simultaneously surviving or not slipping at any of the  $n$  steps,  $[1 - F_w]$  is given by the product of the individual survival probabilities at each of the  $n$  steps. Thus,

$$[1 - F_w] = [1 - F(\mu_c)]^n$$

or

$$F_w = 1 - [1 - F(\mu_c)]^n \quad \text{Eq. 3}$$

If we refer to extreme value statistics [Ref. 16], it turns out that Eq. 3 is the definition of the exact distribution of the smallest value among  $n$  independent observations. This gift of 85 years of mathematical inquiry into the statistics of extremes enables us to state that the precise form of  $F(\mu_c)$  is a Weibull Distribution, i.e.

$$F(\mu_c) = 1 - e^{-\left(\frac{\mu_c - \mu_z}{\mu_o}\right)^m} \dots \mu_c \geq \mu_z \quad \text{Eq. 4}$$

$$= 0 \dots \mu_c \leq \mu_z$$

This result from the asymptotic theory of extreme order statistics was first established in 1928 by Fisher and Tippet [Ref. 17]; it is extensively explored in a remarkable book by Galambos [Ref. 18]. The Weibull form follows from the observations that the friction coefficients are continuously distributed, they are independent and identically distributed,

their distribution is bounded on the left (e.g. zero probability at  $\mu = 0$ ), and a walk of  $n$  steps follows the “weakest-link principle” in the sense that its resistance to slip cannot exceed the lowest friction coefficient encountered.

To find  $F_w$ , Eq. 4 is substituted into Eq. 3,

$$F_w(\mu_c) = 1 - e^{-n \left( \frac{\mu_c - \mu_z}{\mu_o} \right)^m} \quad \dots \mu_c \geq \mu_z \quad \text{Eq. 5}$$

$$= 0 \quad \dots \mu_c \leq \mu_z$$

The Weibull form is recaptured. This simple, elegant formula provides a relationship among the probability of slipping (or falling below  $\mu_c$ ), the length of the walk ( $n$  steps), the critical friction criterion  $\mu_c$ , and three statistical parameters which characterize the floor/footwear set. The three Weibull constants describe the entire distribution of friction coefficients including their average, their spread and their asymmetry. The properties of Eq. 5 are explored in Barnett [Ref. 12].

Some of the characteristics of the new extreme value formulation of the slip problem can be discerned from the sample calculations of slip probability, Eq. 5, presented in Table I. Each of the triplets of Weibull parameters ( $m, \mu_o, \mu_z$ )

Table I - Probability of Slipping: Variation of Parameters

Case	Walk Distance ( $n$ -steps)	Critical Friction Criterion	Weibull Parameters			Average Friction Coefficient	Coefficient of Variation	Probability of Slipping
			$m$	$\mu_o$	$\mu_z$			
No.	$n$	$\mu_c$	$m$	$\mu_o$	$\mu_z$	$\bar{\mu}$	St'd. Dev. $\bar{\mu}$	$F_w(\mu_c)$
1	5	0.33	5	0.207	0.31	0.5	8.71%	$4.21 \times 10^{-5}$
	10	0.33	5	0.207	0.31	0.5	8.71%	$8.42 \times 10^{-5}$
	100	0.33	5	0.207	0.31	0.5	8.71%	$8.42 \times 10^{-4}$
	1000	0.33	5	0.207	0.31	0.5	8.71%	$8.38 \times 10^{-3}$
2	5	0.33	8	0.202	0.31	0.5	5.65%	$4.62 \times 10^{-8}$
	10	0.33	8	0.202	0.31	0.5	5.65%	$9.23 \times 10^{-8}$
	100	0.33	8	0.202	0.31	0.5	5.65%	$9.23 \times 10^{-7}$
	1000	0.33	8	0.202	0.31	0.5	5.65%	$9.23 \times 10^{-6}$
3	5	0.5	5	0.207	0.31	0.5	8.71%	$9.62 \times 10^{-1}$
	10	0.5	5	0.207	0.31	0.5	8.71%	$9.99 \times 10^{-1}$
	100	0.5	5	0.207	0.31	0.5	8.71%	1
	1000	0.5	5	0.207	0.31	0.5	8.71%	1
4	5	0.5	10	0.200	0.31	0.5	4.58%	$9.50 \times 10^{-1}$
	10	0.5	10	0.200	0.31	0.5	4.58%	$9.97 \times 10^{-1}$
	100	0.5	10	0.200	0.31	0.5	4.58%	1
	1000	0.5	10	0.200	0.31	0.5	4.58%	1
5	5	0.33	5	0.425	0.31	0.7	12.77%	$1.15 \times 10^{-6}$
	10	0.33	5	0.425	0.31	0.7	12.77%	$2.31 \times 10^{-6}$
	100	0.33	5	0.425	0.31	0.7	12.77%	$2.31 \times 10^{-5}$
	1000	0.33	5	0.425	0.31	0.7	12.77%	$2.31 \times 10^{-4}$
6	5	0.33	8	0.414	0.31	0.7	8.26%	$1.48 \times 10^{-10}$
	10	0.33	8	0.414	0.31	0.7	8.26%	$2.97 \times 10^{-10}$
	100	0.33	8	0.414	0.31	0.7	8.26%	$2.97 \times 10^{-9}$
	1000	0.33	8	0.414	0.31	0.7	8.26%	$2.97 \times 10^{-8}$
7	5	0.5	5	0.425	0.31	0.7	12.77%	$8.54 \times 10^{-2}$
	10	0.5	5	0.425	0.31	0.7	12.77%	$1.64 \times 10^{-1}$
	100	0.5	5	0.425	0.31	0.7	12.77%	$8.32 \times 10^{-1}$
	1000	0.5	5	0.425	0.31	0.7	12.77%	1
8	5	0.5	10	0.410	0.31	0.7	6.70%	$2.28 \times 10^{-3}$
	10	0.5	10	0.410	0.31	0.7	6.70%	$4.56 \times 10^{-3}$
	100	0.5	10	0.410	0.31	0.7	6.70%	$4.46 \times 10^{-2}$
	1000	0.5	10	0.410	0.31	0.7	6.70%	$3.37 \times 10^{-1}$

shown in Table I represents a floor/footwear couple; there are actually five different sets shown. Associated with each triplet is an assumed critical friction criterion  $\mu_c$  and a walk distance expressed in number of steps  $n$  walked by a pedestrian. This information, together with Eq. 5, enables the calculation of  $F_w(\mu_c)$  which provides the probability of pedestrians encountering friction coefficients with values equal to or below the critical friction criterion  $\mu_c$ . These probabilities of slipping are tabulated in Table I together with the average and coefficient of variation corresponding to the Weibull triplet. Expressions for the mean and standard deviation may be found in Ref. 12, Eqs. 6 and 7 respectively. It should be noted that two values of  $\mu_c$  are considered in Table I;  $\mu_c = 0.5$  is the most popular criterion and  $\mu_c = 0.33$  is related to force-plate testing.

Observe that in all cases described in Table I,  $\bar{\mu} \geq \mu_c$ . Classical theory incorrectly predicts no slipping. Most real floors infrequently violate  $\mu_c = 0.33$  which is reflected by the low slip probabilities shown in Cases 1, 2, 5, and 6. In these cases an order of magnitude increase in the number of steps  $n$  correspondingly increases the fall probabilities by an order of magnitude. Comparing Cases 1 and 2 one finds that when all things are equal, the smaller scatter (lower coefficient of variation) associated with Case 2 leads to a smaller  $F_w(\mu_c = 0.33)$ . A similar comparison among equals may be made between Cases 1 and 6. Here, the scatter is almost equal, but  $\bar{\mu} = 0.7$  for Case 6 and  $\bar{\mu} = 0.5$  for Case 1. Under these circumstances, the average friction dominates slip behavior; Case 6 has the higher slip resistance and the lower  $F_w(\mu_c = 0.33)$ .

As a final comparison consider Cases 2 and 5 where the lower slip probabilities are associated with Case 2 which has an average friction coefficient of  $\bar{\mu} = 0.5$ . The higher friction floor in Case 5 has  $\bar{\mu} = 0.7$ ; it produces a great many more slips. Not only does this result unsettle one of our stalwart notions, it implies that the average friction coefficient cannot even be used for ranking floors.

### III. Duty Cycle - Homogeneous and Isotropic Floors

A floor will be defined as homogeneous if it exhibits the same statistical behavior at every location. Further, a floor will be defined as isotropic if it exhibits the same statistical behavior in every direction in the plane of the floor. A walk profile on a homogeneous and isotropic floor may be characterized by the number of steps  $n_j$  taken by an individual during a particular ambulation. Let  $T_j$  be the total number of walk profiles consisting of exactly  $n_j$  steps. The duty cycle for the floor may then be defined as the entire collection of walk profiles undertaken by pedestrians during a specified time period. It may be designated as

$$(n_j, T_j) \text{ where } j = 1, 2, \dots, \beta$$

and where  $\beta$  is the total number of different walk profiles.

As an example, a floor's duty cycle may be determined using a security camera which monitors the walking activities on a

single, homogeneous, and isotropic floor during a two-day period. Counting the number of steps for each pedestrian during this surveillance, a set of walking profiles may be recorded as shown in the sample displayed in Table II. A tally chart of this data is shown in Table III where the number of steps  $n_j$  (walking profile) is ordered from the smallest to the largest value in column 1. The observed tallies in column 2 are summed and listed in the third column which shows the absolute frequency with which each similar walk profile occurs.

Table II - Sample of 50 Walking Profiles - Number of Steps,  $n_j$ .

6	10	8	18	9
8	7	10	5	8
13	8	11	12	16
4	8	5	17	12
10	7	9	17	9
12	8	13	11	18
8	5	16	4	8
10	11	4	12	16
7	13	8	9	8
6	8	4	8	14

Time Period: 2 days

Table III - - Duty Cycle;  $(n_j, T_j)$  or  $(n_j, T_j/T)$

Number of Steps (Walking Profiles)	Absolute Frequency *		Relative Frequency **
	Tallies	Number of Walk Profiles	$\beta = 15$ $T \equiv \sum_{j=1} T_j = 50$
$n_j$	Tallies	$T_j$	$T_j/T$
$n_1 = 4$	IIII	4	0.08
$n_2 = 5$	III	3	0.06
$n_3 = 6$	II	2	0.04
$n_4 = 7$	III	3	0.06
$n_5 = 8$	IIII IIII II	12	0.24
$n_6 = 9$	IIII	4	0.08
$n_7 = 10$	IIII	4	0.08
$n_8 = 11$	III	3	0.06
$n_9 = 12$	IIII	4	0.08
$n_{10} = 13$	III	3	0.06
$n_{11} = 14$	I	1	0.02
$n_{12} = 15$		0	0
$n_{13} = 16$	III	3	0.06
$n_{14} = 17$	II	2	0.04
$n_{15} = 18$	II	2	0.04

\* Time Period: 2 Days

\*\* Relative Frequencies are independent of time period

The first and third columns describe the floor's duty cycle  $(n_j, T_j)$  for a two-day time period. This duty cycle may be conveniently characterized using the bar graph of absolute frequencies illustrated in Fig. 2-a. This duty cycle represents the floor usage pattern for the specified time period, two-days.

There are  $\beta$  possible walk profiles which give rise to a total number of pedestrian ambulations  $T$  where

$$T = \sum_{j=1}^{\beta} T_j, \quad j = 1, 2, \dots, \beta$$

in a specified time period. The fraction  $T_j/T$  provides the proportion of pedestrians who walk exactly  $n_j$  steps. This fraction also represents the relative frequency shown in column 4 of Table III for each walk profile. Since  $T_j$  and  $T$  represent the same time period, the relative frequency  $T_j/T$  is independent of any specified surveillance period. The veracity of the duty cycle characterized by  $(n_j, T_j/T)$  depends on how well the surveillance period represents the floor usage. Alternatively, the total number of pedestrians studied,  $T$ , must be sufficiently large to accurately represent the floor usage pattern.

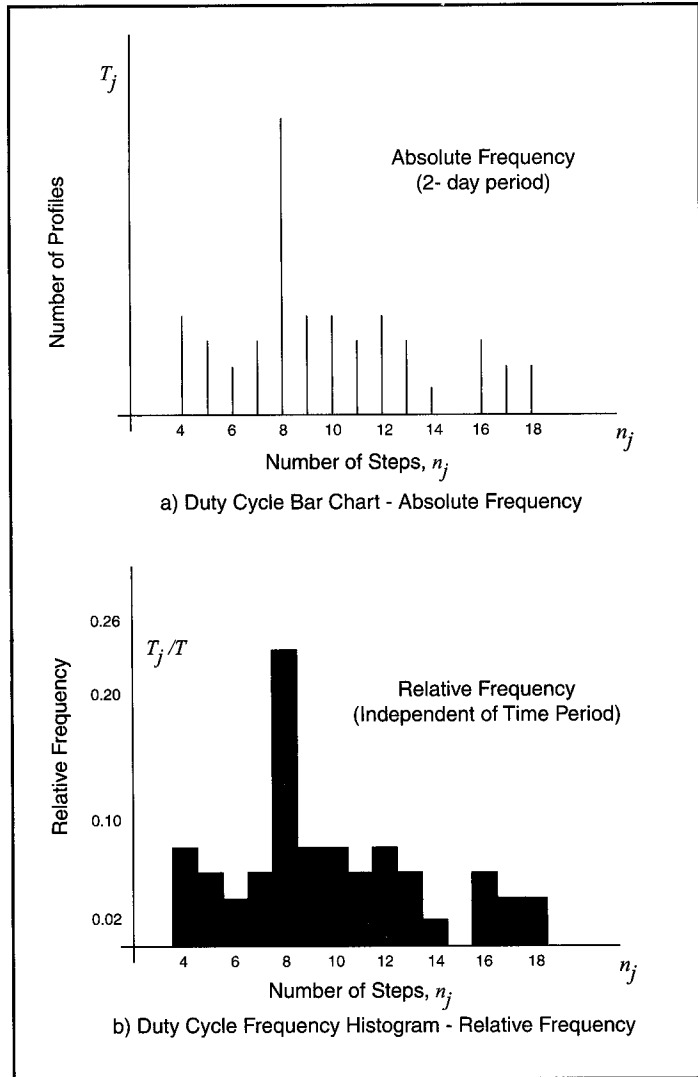


Figure 2 - Duty Cycle Diagrams of Sample Shown in Table II

The duty cycle  $(n_j, T_j/T)$  may be conveniently presented as a frequency histogram [Ref. 14] which is independent of any time frame. The histogram depicted in Fig. 2-b has been constructed using columns 1 and 4 in Table III.

## V. Evaluation of a Homogeneous and Isotropic Floor

A floor is characterized by establishing the number or percentage of pedestrians who will slip while negotiating a given duty cycle. The probability of slipping during a walk of  $n_j$  steps,  $F_w(n_j)$ , is given by Eq. 5 for a particular floor/footwear couple and critical slip criterion  $\mu_c$ . If  $T_j$  pedestrians undertake such a walk, the number of slips is given by  $T_j F_w(n_j)$ . The total number of walkers who will slip during the floor's duty cycle is found by adding the slips associated with each of the  $\beta$  walk profiles, i.e.,

$$\text{Total Number of Slips} = \sum_{j=1}^{\beta} T_j F_w(n_j) \quad \text{Eq. 7}$$

The fraction of pedestrians who will slip during a duty cycle is found by dividing both sides of Eq. 7 by the total number of pedestrians  $T$ ; thus,

$$\begin{aligned} \text{Percentage of slips} &= 100 \left[ \left( \frac{T_1}{T} \right) F_w(n_1) + \left( \frac{T_2}{T} \right) F_w(n_2) + \dots \right. \\ &\left. + \left( \frac{T_\beta}{T} \right) F_w(n_\beta) \right] = 100 \sum_{j=1}^{\beta} \left( \frac{T_j}{T} \right) F_w(n_j) \quad \text{Eq. 8} \end{aligned}$$

Table IV displays all of the calculations relative to the evaluation of the asphalt floor characterized in Fig. 1 with a critical friction criterion  $\mu_c = 0.5$  which is subjected to the duty cycle portrayed in Table III. We observe that twelve of the fifty pedestrians who traversed the asphalt floor in the two-day surveillance period slipped or, more accurately, encountered a friction coefficient equal to or smaller than  $\mu_c$ . The last column of Table IV indicates that the floor's slip rate is 24.29% for its duty cycle, i.e., 243 people out of a thousand will slip or engage a friction coefficient equal to or below the critical friction  $\mu_c$ .

## V. CONCLUSIONS

- The average coefficient of friction has no physical meaning because the bell shaped frequency distribution,  $f(\mu)$ , is asymmetric. The most probable value of  $\mu$  is given by its mode.
- When  $\bar{\mu} > \mu_c$ , conventional theory predicts that no slipping will occur. The new theory recognizes that slipping will transpire under these conditions at a rate given by  $F_w(\mu_c)$  for a single ambulation.
- A floor/footwear couple with a high average friction coefficient may produce more slips than one with a lower average. The average friction coefficient  $\bar{\mu}$  cannot be used to rank floor surfaces.

Table IV - Slip Calculations for Duty Cycle Described in Table III

Walk Profile	Number of Steps	Number of Pedestrians	Slip Probability	Number of Slips (2-day period)	Slip Proportion
$j$	$n_j$	$T_j$	$F_w(n_j) = 1 - e^{-n_j} \left( \frac{0.5 - 0.31}{0.40} \right)^{4.75}$	$T_j F_w(n_j)$	$(T_j / T) F_w(n_j)$
1	4	4	0.10998	0.43991	$8.798 \times 10^{-3}$
2	5	3	0.13553	0.40658	$8.132 \times 10^{-3}$
3	6	2	0.16034	0.32069	$6.414 \times 10^{-3}$
4	7	3	0.18445	0.55334	$11.067 \times 10^{-3}$
5	8	12	0.20786	2.49430	$49.886 \times 10^{-3}$
6	9	4	0.23060	0.92239	$18.448 \times 10^{-3}$
7	10	4	0.25269	1.01074	$20.215 \times 10^{-3}$
8	11	3	0.27414	0.82242	$16.448 \times 10^{-3}$
9	12	4	0.29498	1.17990	$23.598 \times 10^{-3}$
10	13	3	0.31521	0.94564	$18.913 \times 10^{-3}$
11	14	1	0.33487	0.33487	$6.697 \times 10^{-3}$
12	15	0	0.35397	0	0
13	16	3	0.37251	1.11754	$22.351 \times 10^{-3}$
14	17	2	0.39053	0.78105	$15.621 \times 10^{-3}$
15	18	2	0.40802	0.81604	$16.321 \times 10^{-3}$
		Total 50	Critical Friction Criterion $\mu_c = 0.5$	Total 12.145	Total $242.9 \times 10^{-3}$

- D. The reliability of a floor/footwear couple during a single ambulation is expressed as  $[1 - F_w(\mu_c)]$ . Using Eq. 5 this may be written as

$$\left[1 - F_w(\mu_c)\right] = e^{-n \left(\frac{\mu_c - \mu_z}{\mu_o}\right)^m} \quad \text{Eq. 9}$$

Clearly, the reliability decreases as the number of steps  $n$  increases. A floor that has a perfectly satisfactory reliability for short walks  $n$  may give rise to too many slips during long ones  $N$ .

- E. The bell shaped probability density curve that defines the distribution of friction coefficients is a Weibull Distribution.
- F. Conventional slip theory uses only the central measure of the coefficient of friction distribution; specifically, its mean  $\bar{\mu}$ . The new theory given by Eq. 5 reflects the central measure, the scatter, and the asymmetry of the probability density distribution representing the friction coefficients.
- G. An examination of Table I reveals that the average friction  $\bar{\mu}$  does not characterize a floor. Cases 5 and 6 or Cases 7 and 8 show widely varying slip probabilities for the same average friction  $\bar{\mu} = 0.7$ . A floor preparation may, for example, greatly improve a floor/footwear couple without changing the mean friction coefficient.
- H. To evaluate the percentage of people who will slip on a real floor, it is necessary to input its usage pattern so that Eq. 5 may be applied. The floor usage may be characterized by establishing its duty cycle.
- I. For an isotropic and homogenous floor the duty cycle can be represented as a standard frequency histogram.
- J. Mixed footwear and directional floor properties give rise to nonhomogenous and anisotropic floors which will be treated in a future publication. In the meantime, a worst case floor/footwear couple will provide a lower bound on the floor reliability.

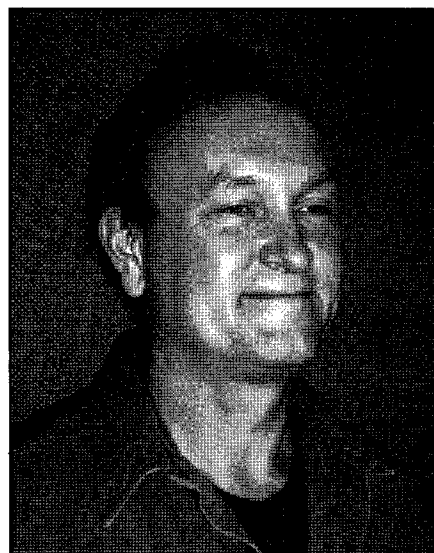
## V. References

1. Adler, S.C. & Pierman, B.C. (1979). *History of walkway slip-resistance research at the National Bureau of Standards* (Report No. NBS SP 505). Washington, D.C.: National Bureau of Standards.
2. American Society for Testing and Materials (ASTM). (1983b). *Proposed test method for static coefficient of friction of polish-coated floor surfaces as measured by the NBS-Brungraber Articulated Arm Tester*. (Standard No. ASTM Proposed Standard P126). Philadelphia, Pennsylvania: Author.
3. American Society for Testing and Materials (ASTM). (1983c). *Proposed test method for dynamic coefficient of friction of polish-coated floor surfaces as measured by the NBS-Sigler Pendulum Impact Tester*. (Standard No. ASTM Proposed Standard P127). Philadelphia, Pennsylvania: Author.
4. American Society for Testing and Materials (ASTM). (1983d). *Proposed test method for static and dynamic coefficient of friction of polish-coated floor surfaces as measured by the Topaka Slip Tester*. (Standard No. ASTM Proposed Standard P128). Philadelphia, Pennsylvania: Author.
5. American Society for Testing and Materials (ASTM). (1988b). *Standard test method for static coefficient of friction of shoe sole and heel materials as measured by the James Machine*. (Standard No. ASTM F489-77 [Reapproved 1988]). Philadelphia, Pennsylvania: Author.
6. Balance, P.E., Morgan, J., & Senior, D. (1985). Operational experience with a portable friction testing device in university buildings. *Ergonomics*, 28, 1043-1054.
7. Brungraber, R.J. (1977). A new portable tester for the evaluation of the slip-resistance of walkway surfaces. *NBS Technical Note 953*. Washington, D.C.: National Bureau of Standards.
8. Jung, K. & Schenk, H. (1990). An international comparison of test methods for determining the slip resistance of shoes. *Journal of Occupational Accidents*, 13 (4), 271-290.
9. Majcherczyk, R. (1977). Different approach to measuring pedestrian friction: The CEBTP skidmeter. In C. Anderson & J. Senne (Eds.), *Walkway Surfaces: Measurements of Slip Resistance*, (Publication No. ASTM STP 649). (pp. 88-99). Philadelphia, Pennsylvania: American Society for Testing and Materials.
10. Pfauth, M.J. & Miller, J.M. (1976). Work surface coefficients: A survey of relevant factors and measurement methodology. *Journal of Safety Research*, 8 (2), 77-90.

11. Reed, M.E. & Mahon, R.D. (1977). Description of the National Institute for Occupational Safety and Health (NIOSH) universal friction testing machine (UFTM). In C. Anderson & J. Senne (Eds.), *Walkway Surfaces: Measurement of Slip Resistance*, (Publication No. ASTM STP 649). (pp. 60-70). Philadelphia, Pennsylvania: American Society for Testing and Materials.
12. Barnett, Ralph L. " 'Slip and Fall' Theory - Extreme Order Statistics," *International Journal of Occupational Safety and Ergonomics*, Vol. 8, No. 2, 2002.
13. "Standard Test Method for Static Slip Resistance for Footwear, Sole, Heel, or Related Materials by Horizontal Pull Slipmeter (HPS)," *ASTM F609-79 (Reapproved 1989)*. Philadelphia: American Society for Testing and Materials, 1989.
14. Kreyszig, Erwin. *Advanced Engineering Mathematics*. New York: John Wiley & Sons, 1999. p. 1106.
15. Hoel, Paul G. *Introduction to Mathematical Statistics*. New York: John Wiley & Sons, Fourth Edition, 1971. p. 15.
16. Gumbel, E.J. *Statistics of Extremes*. New York: Columbia University Press, 1958.
17. Fisher & Tippett, "The Frequency Distribution of the Largest or Smallest Member of a Sample," *Proceedings of Cambridge Philosophical Society* 24, 1928. pp. 180-190.
18. Galambos, J. *The Asymptomatic Theory of Extreme Order Statistics*. New York: John Wiley & Sons, 1978.

## Triodyne Tech Earns Another Master Certification!

Triodyne would like to congratulate Charles Sinkovits on becoming an ASE Certified Master School Bus Technician. Charly is also a Certified Master Auto Technician and a Certified Master Truck Technician. Only 493 techs or 0.1% of all the technicians in the country, have all three Master certifications. Congratulations to Charly on joining this elite group.



*Charly Sinkovits*

## **SAFETY BRIEF**

February, 2003 – Volume 22, No. 4

**Editor: Paula L. Barnett**

**Illustrated and Produced by  
Triodyne Graphic Communication**

Copyright, 2003 Triodyne, Inc. All Rights Reserved. No portion of this publication may be reproduced by any process without written permission of Triodyne, Inc., 5950 West Touhy Avenue, Niles, IL 60714-4610 (847) 677-4730. Direct all inquiries to: *Library Services*.