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SLOPED SURFACES - LADDER SLIDE OUT

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ABSTRACT

Straight and extension ladders are designed to be operated on firm, level surfaces where resistance to ladder slide out is provided almost entirely by the friction resistance between the ladder foot and the base surface. The required friction resistance for a ladder on a sloped surface increases dramatically as the slope away from the vertical support structure becomes steeper. To maintain the current slide out resistance specified by ladder standards, the ladder inclination must be increased by the ground slope.

I. INTRODUCTION

The geometry and general loading of typical straight, combination, and extension ladders on a level surface is shown in Fig.1 for an angle of inclination θ. In the U.S., the recommended angle of inclination θ^* corresponds to a ladder set-up with $c/L = 1/4$ or,

$$\theta^* = \cos^{-1}\frac{1}{4} = 75.52° \qquad \text{Eq. 1}$$

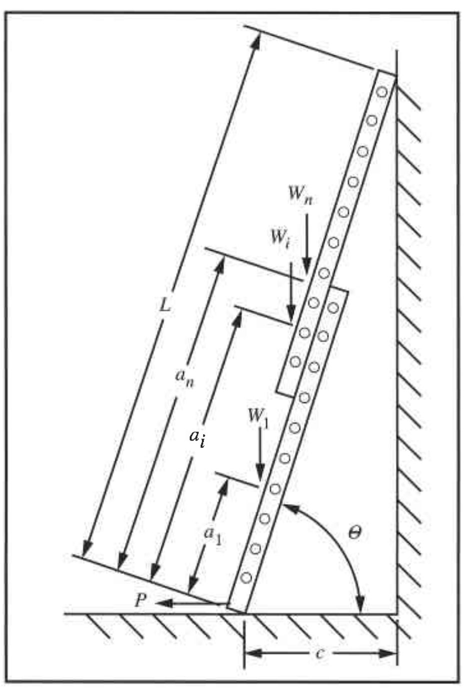

Figure 1 - Ladder Geometry and General Loading

No Charge

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To stabilize the ladder described in Fig. 1 against slide out, the coefficient of friction for the ladder foot/base couple has been established in Barnett 1999 as

$$\mu_b \geq \frac{(\mu_t + \tan\theta)\dfrac{P}{W} + \sum_{i=1}^{n}\left(\dfrac{W_i}{W}\right)\left(\dfrac{a_i}{L}\right)}{(\mu_t + \tan\theta) - \mu_t\sum_{i=1}^{n}\left(\dfrac{W_i}{W}\right)\left(\dfrac{a_i}{L}\right)} \text{ ... no slide out , Eq.2}$$

where

μ_b ... coefficient of friction between the base and the ladder foot.

μ_t... coefficient of friction between the wall and the ladder top.

W... total weight of gravity loading, i.e.,

$$W = \sum_{i=1}^{n} W_i$$

W_i... i^{th} gravity force.

a_i... location of the i^{th} gravity force measured from the ladder base.

L... extended ladder length.

θ... angle of ladder inclination.

P... horizontal pulling force – Foot Slip Tests, ANSI A14.2.

This relationship assumes a firm, level base surface in accordance with all current admonitions promulgated by ladder manufacturers and ladder standard ANSI A14.2-2000.

The objective of this study is to describe the required base friction coefficient when the ground surface slopes downward from the support wall. The free body diagram depicted in Fig. 2 is used to develop three equations of planar equilibrium for the sloped ground. It should be noted that the various gravity loads W_i have been replaced by the total load W acting at their center of force $\bar{a}$. By definition the center of force produces the same moment about the base, $W\bar{a}\cos\theta$, as the sum of the n individual gravity force moments; thus,

$$\bar{a} = \sum_{i=1}^{n}\left(\frac{W_i}{W}\right)a_i \qquad \text{Eq. 3}$$

Taking the sum of forces in the vertical and horizontal directions and the moment about the ladder base, we obtain respectively the following equilibrium equations:

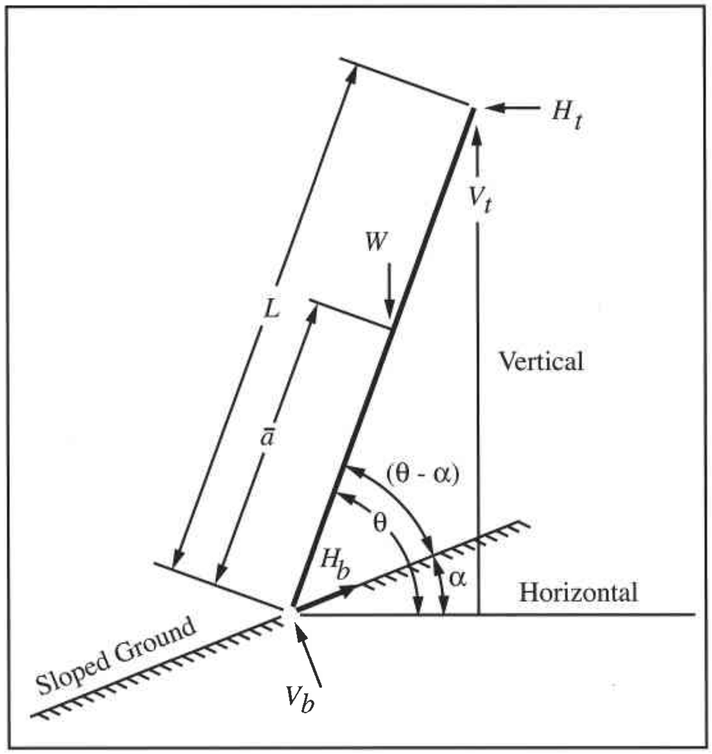


Figure 2 - Free Body Diagram

$$W - V_t - V_b\cos\alpha - H_b\sin\alpha = 0 \qquad \text{Eq. 4}$$

$$H_t - H_b\cos\alpha + V_b\sin\alpha = 0 \qquad \text{Eq. 5}$$

$$W\bar{a}\cos\theta - V_t L\cos\theta - H_t L\sin\theta = 0 \qquad \text{Eq. 6}$$

where α is the surface declination defined in Fig. 2. These equations may be solved for V_t, V_b, and H_t in terms containing the unknown reaction H_b; thus,

$$V_t = \frac{W\left(\dfrac{\bar{a}}{L}\cos\alpha + \tan\theta\sin\alpha\right) - H_b\tan\theta}{\cos(\theta-\alpha)/\cos\theta} \qquad \text{Eq. 7}$$

$$V_b = \frac{W\left(1 - \dfrac{\bar{a}}{L}\right) + H_b[\sin(\theta-\alpha)/\cos\theta]}{\cos(\theta-\alpha)/\cos\theta} \qquad \text{Eq. 8}$$

$$H_t = \frac{-W\left(1 - \dfrac{\bar{a}}{L}\right)\sin\alpha + H_b}{\cos(\theta-\alpha)/\cos\theta} \qquad \text{Eq. 9}$$

When a ladder is on the brink of sliding out, the top contact is in a condition of incipient slip. Consequently, the vertical reaction V_t may be written as the product of the normal wall force H_t and the coefficient of friction μ_t between the wall and the top of the ladder, i.e.,

$$V_t = \mu_t H_t \qquad \text{Eq. 10}$$

Substituting Eqs. 7 and 9 into Eq. 10 provides the following expression for H_b:

$$H_b = W\left\{\frac{\sin\alpha\left[\mu_t\left(1-\frac{\bar{a}}{L}\right)+\tan\theta\right]+\frac{\bar{a}}{L}\cos\alpha}{\mu_t+\tan\theta}\right\} \qquad \text{Eq. 11}$$

This relationship for H_b may be used to uniquely determine the reactions V_t, V_b, and H_t by substituting Eq. 11 into Eqs. 7, 8, and 9 respectively.

Other than H_b, only the reaction V_b is required to study the slip propensity of the ladder base; thus,

$$V_b = \frac{W\left(1-\frac{\bar{a}}{L}\right)\cos\theta}{\cos(\theta-\alpha)} + W\tan(\theta-\alpha)\left\{\frac{\sin\alpha\left[\mu_t\left(1-\frac{\bar{a}}{L}\right)+\tan\theta\right]+\frac{\bar{a}}{L}\cos\alpha}{\mu_t+\tan\theta}\right\} \qquad \text{Eq. 12}$$

The base of a ladder cannot slip if the reaction H_b does not exceed the incipient slip resistance $\mu_b V_b$ where μ_b is the coefficient of friction between the sloped surface and the ladder feet, i.e.,

$$H_b \le \mu_b V_b \ldots \text{ no slip}$$

or

$$\mu_b \ge \frac{H_b}{V_b} \ldots \text{ no slip} \qquad \text{Eq. 13}$$

Using Eqs. 11 and 12, Eq. 13 becomes the "no slip criterion" for sloped surfaces;

$$\mu_b \ge \frac{1}{\tan(\theta-\alpha)+\dfrac{\left(1-\frac{\bar{a}}{L}\right)(\mu_t\cos\theta+\sin\theta)}{\cos(\theta-\alpha)\left\{\sin\alpha\left[\mu_t\left(1-\frac{\bar{a}}{L}\right)+\tan\theta\right]+\frac{\bar{a}}{L}\cos\alpha\right\}}} \qquad \text{Eq. 14}$$

II. NO SLIP CRITERION

A. Level Surface: $\alpha = 0$

Taking $\alpha = 0$ in Eq. 14, we obtain

$$\mu_b \ge \frac{\left(\frac{\bar{a}}{L}\right)}{(\mu_t+\tan\theta)-\mu_t\frac{\bar{a}}{L}} \qquad \text{Eq. 15}$$

Equation 15 is precisely Eq. 2 with the applied test force $P = 0$.

B. Frictionless Walls: $\mu_t = 0$

Sloped surface:

When $\mu_t = 0$, Eq. 14 becomes

$$\mu_b \ge \frac{1}{\tan(\theta-\alpha)+\dfrac{\left(1-\frac{\bar{a}}{L}\right)\sin\theta}{\cos(\theta-\alpha)\cos\alpha\left(\tan\alpha\tan\theta+\frac{\bar{a}}{L}\right)}} \qquad \text{Eq. 16}$$

Level Surface:

Using Eq. 2 or Eq. 15, we obtain a simple expression for the "no slip criterion;"

$$\mu_b \ge \frac{(\bar{a}/L)}{\tan\theta} \qquad \text{Eq. 17}$$

C. Critical Loading: $(\bar{a}/L) = 1$

When gravity loads are concentrated at the top of a ladder, $(\bar{a}/L) = 1$, the base friction coefficient μ_b required to prevent ladder slide out assumes its greatest value. This may be inferred from Eq. 2 or Eq. 14; it is discussed extensively in Barnett 1999. When $(\bar{a}/L) = 1$ is substituted in Eq. 14, the no slip criterion is simply,

$$\mu_b \geq \frac{1}{\tan(\theta - \alpha)} \qquad \cdots(\bar{a}/L) = 1 \qquad \text{Eq. 18}$$

For level surfaces, Eq. 2 and Eq. 18 provide

$$\mu_b \geq \frac{1}{\tan\theta} \qquad \cdots(\bar{a}/L) = 1 \qquad \text{Eq. 19}$$

The minimum value of base friction μ_b* required to prevent slide out on a level surface with a ladder inclination $\theta = \theta^*$ and $(\bar{a}/L) = 1$, is given by the equality in Eq. 19,

$$\mu_b{}^* = \frac{1}{\tan\theta^*} \qquad \cdots(\bar{a}/L) = 1 \qquad \text{Eq. 20}$$

To provide equivalent protection on a sloped surface for which $\mu_b = \mu_b{}^*$, the ladder inclination θ with the horizontal direction (see Fig. 2) must be set so that the equalities in Eqs. 18 and 20 are equal to each other; thus,

$$\theta = \theta^* + \alpha \quad \cdots(\bar{a}/L) = 1 \qquad \text{Eq. 21}$$

What could be simpler? Add the ground slope α to θ^* to obtain the ladder inclination which preserves $\mu_b{}^*$. Unfortunately, there is physically no straightforward method for setting the inclination $(\theta^* + \alpha)$ on a real ladder.

A vertical ladder inclination $\theta = 90°$ is clearly a limiting condition; indeed, real ladders will fall over backwards when the inclination is too steep. Consequently, the method of adjusting a ladder's inclination corresponding to the minimum base friction $\mu_b{}^*$ is limited to $\theta = (\theta^* + \alpha) \leq 90°$. Thus, if $\theta^* = 75.52°$, the base slope α must not exceed $14.48°$. When $\theta = \theta^* = 75.52°$, $\mu_b{}^* = 1/\tan 75.52° = 0.258$. For base slopes steeper than $14.48°$, non-slip requires base friction between the ladder foot and the base surface to exceed $\mu_b{}^*$ for any angle $\theta \leq 90°$.

Under worst case loading, $(\bar{a}/L) = 1$, the minimum base friction $\mu_b = \mu_b{}^{**}$ required to prevent slipping on a base slope α is given by the equality in Eq. 18,

$$\mu_b{}^{**} = 1/\tan(\theta - \alpha) \qquad \ldots(\bar{a}/L) = 1 \qquad \text{Eq. 22}$$

This relationship is tabulated in Table I for $\theta = \theta^* = 75.52°$ to indicate the sensitivity of $\mu_b{}^{**}$ with increasing ground slope.

To achieve a non-slip condition on the worst possible base slope, one would use a vertical ladder inclination $\theta = 90°$ and the largest conceivable base friction $\mu_b{}^{**} = 1$.

Table I - Minimum "No Slip Base Friction": $(\bar{a}/L) = 1$

Base Slope, α (degrees)	Base Friction $\mu_b = 1/\tan(75.52° - \alpha)$
0	0.258
1	0.277
2	0.296
3	0.315
4	0.334
5	0.354
6	0.373
7	0.394
8	0.414
9	0.434
10	0.455
14.48	0.553
20	0.687
25	0.824

Using these parameters in Eq. 18 gives

$$1 = 1/\tan(90° - \alpha) \Rightarrow \alpha = 45°$$

Consequently, ladder applications will always be restricted to base slopes less than $45°$.

When a ladder is misused by installing it on a sloped surface with the traditional inclination $\theta = \theta^*$, its safety may be judged by examining the base friction levels required by ANSI A14.2-2000 which reflect the specific minimum coefficients of friction between ladder feet and the A surface of an A-C plywood panel which is presanded using 320 fine wet/dry sandpaper. These friction criteria are displayed in Table II; they are calculated by applying Eq. 2 to a standardized test protocol as discussed in Barnett 1999. It would appear from Table I and Table II that the safety factors incorporated in the ANSI standard will safely equilibrate ladders on small slopes without increasing the ladder inclination with the horizontal direction above $\theta = 75.52°$.

III. ANTI-SLIDE OUT INVENTION

An automatic device has recently been patented by Barnett 2002 for erecting straight ladders at the traditional inclination $\theta = \theta^* = 75.52°$ on a level surface. The invention, which is described in an ASME paper by Switalski and Barnett 2003, is depicted in Fig. 3 in its most basic form. Figure 3a illustrates how one leg of a rigid triangular ladder base forms an angle θ^* with the level surface. At inclination

Table II - Required Base Friction

Duty Rating ANSI Type	Minimum Base Friction (includes safety factor) μ^*_b
Extra Heavy Duty Type IA	0.380
Heavy Duty Type I	0.409
Medium Duty Type II	0.430
Light Duty Type III	0.455

16 ft. Aluminum Extension Ladders (ANSI A14.2 - 2000)

angles less than θ^*, a pair of wheels located on the inboard vertex of the triangular base ride on the support surface causing the ladder to "roll out" from the wall as shown in Fig. 3b. This makes it impossible to set up the ladder at inclinations shallower than θ^*. On the other hand, at inclinations greater than θ^*, the wheels are lifted from the ground as illustrated in Fig. 3c. This steeper angle improves the slide out resistance of the ladder while compromising its resistance to overturning when a climber leans back.

When a ladder using the triangular base device is properly erected on a sloped surface, as indicated in Fig. 4, the ladder inclination θ is precisely $\theta^* + \alpha$. According to Eq. 21, this provides the same slide out resistance achieved on a level surface. It is entirely serendipitous that the triangular base invention compensates automatically for sloped surfaces.

IV. OBSERVATIONS

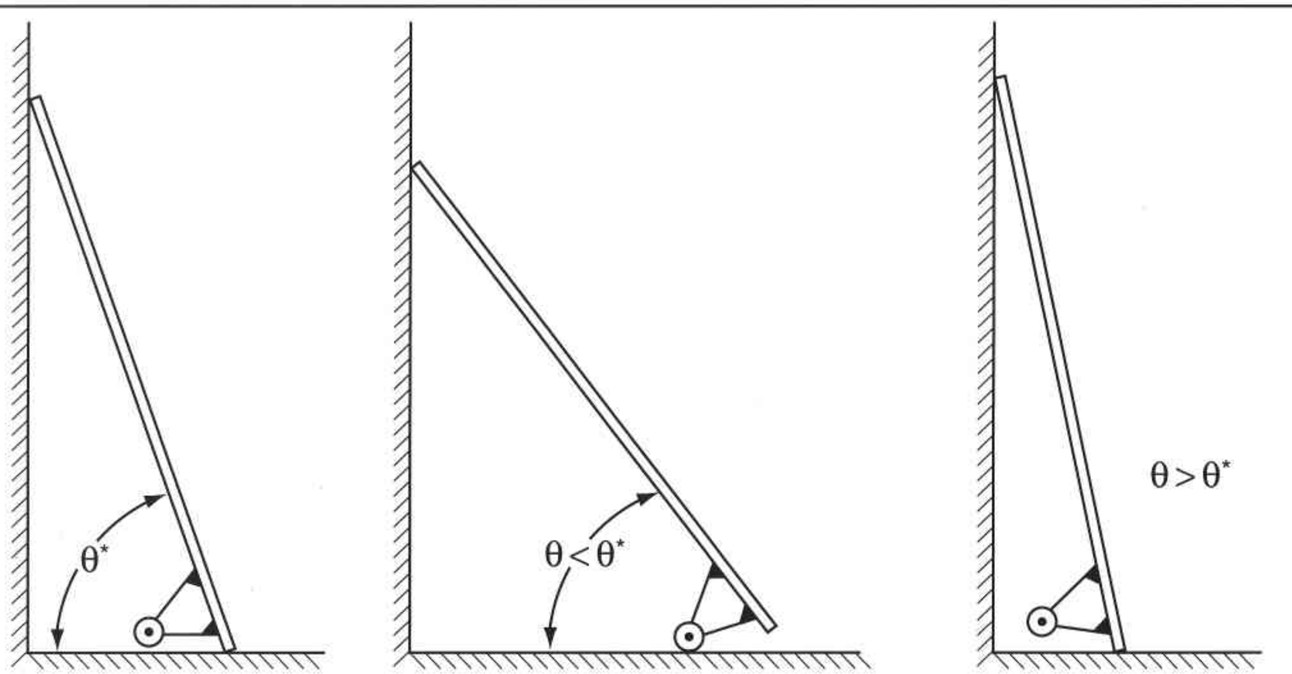
1. The base frictions required to prevent ladder slide out on sloped surfaces are described by Eq. 14. These friction coefficients are independent of the weight of the ladder and climbers; only the weight distribution is important, $(\bar{a}/L)$.

2. The most critical ladder loading occurs when $(\bar{a}/L) = 1$; the loading is concentrated at the very top of the ladder. Realistic critical loadings always give rise to load centers somewhat less than unity.

3. At the worst case loading, $(\bar{a}/L) = 1$, Eq. 14 reduces to the simple form,

$$\mu_b \geq 1/\tan(\theta - \alpha) \qquad \ldots \text{ no slip}$$

Vertical ladder inclinations cannot be used, i.e., $\theta < 90^\circ$.



a) Optimum Inclination | b) Too Shallow (Roll-Away) | c) Too Steep (No Slide Out; May Overturn)

Figure 3 - Triangular Base Device - Level Surface

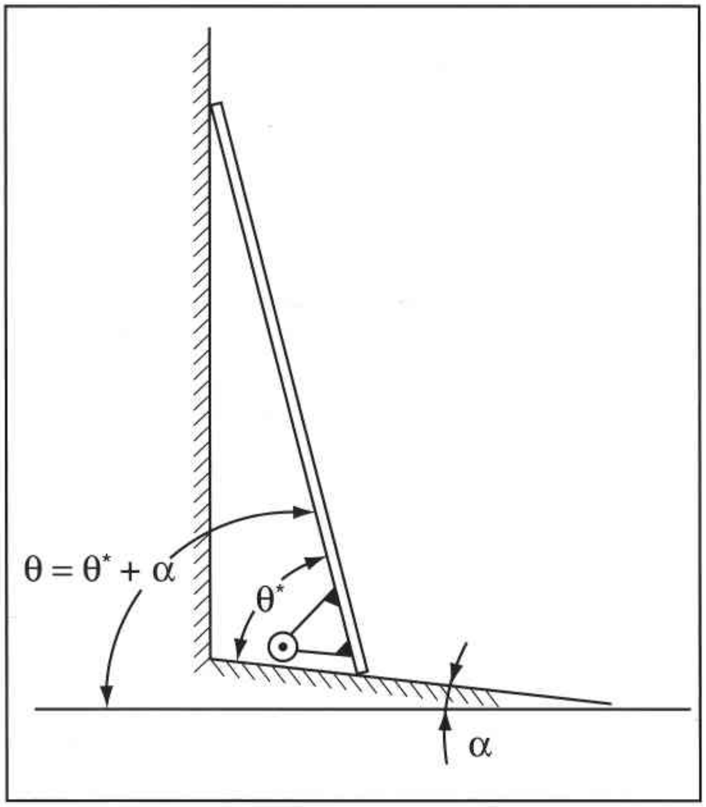


Figure 4 - Triangular Base Device - Sloped Surface

4. To compensate for a sloped surface, the ladder inclination may be increased from θ to $\theta + \alpha$. When using the recommended inclination θ^*, the set up angle $\theta^* + \alpha$ may become so steep that climbers will overturn the ladder.

5. The friction safety factors adopted by the ANSI standards will easily tolerate small deviations from a level support surface.

6. The compromising effect of a sloped surface combined with users' natural tendency to use shallow inclinations may lead to devastating slide out scenarios. According to the findings of the Liberty Mutual Insurance Company 1977 for 16-foot aluminum extension ladders, the intuitive setup methods homeowners described gave rise to an average set up inclination of $68.25°$ with a standard deviation of $\sigma = 3.86°$. Assuming a normal frequency distribution, 99.7% of the angles selected by intuition will fall in the range,

$$56.67° \leq \theta \leq 79.83°$$

This unacceptable range is further compromised by ground slopes.

7. There are many methods currently employed for establishing a ladder inclination of $\theta = \theta^* = 75.52°$. All of them are ultimately based on a plumb reference axis. When confronted with a sloped surface, a compensating ladder inclination of $\theta = 75.52° + \alpha$ cannot be set using any of the known concepts. On the other hand, the new invention using a triangular ladder base automatically accommodates the sloped surface by using the surface itself as the reference. This maintains the non-slip friction resistance for all base slopes.

REFERENCES

1. Barnett, R.L., "Ladder Slide Out"– First Order Analysis,""DETC99/RSAFP-8865, Las Vegas, NV, American Society of Mechanical Engineers Design and Engineering Technical Conference, September, 1999.

2. American National Standard Institute, ANSI A14.2 - 2000.

3. Barnett, R.L., "Anti-Slide Out Devices For Straight and Extension Ladders," U.S. Patent No. 6,405,829 B1 issued June 18, 2002.

4. Switalski, William G. and Barnett, Ralph L., "Auto-Setting Ladder Inclination," DETC03/SERA-48689, Chicago, IL, American Society of Mechanical Engineers, September 2003.

5. Irmin, Charles H. and Marian Vejvodea, "An Investigation of the Angle of Inclination for Setting Non-self-supporting Ladders," Professional Safety 22, No. 7, Liberty Mutual, July 1977.

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