

Slipping on Concrete: A Case Study

Ralph L. Barnett

Triodyne Inc.
450 Skokie Boulevard, Suite 604
Northbrook, IL 60062
(847) 677-4730, Fax (847) 647-2047
infoserv@triodyne.com

Adam A.E. Ziemba

SRI International
333 Ravenswood Avenue
Menlo Park, CA 94025
(650) 859-2353, Fax (650) 859-2343
adam.ziemba@sri.com

Theodore Liber

Triodyne Inc.
450 Skokie Boulevard, Suite 604
Northbrook, IL 60062
(847) 677-4730, Fax (847) 647-2047
ted@triodyne.com

ABSTRACT

The notion of slipperiness is rarely associated with a concrete walkway. The aggressive nature of this surface invariably satisfies the classical criterion of a safe floor. The case study described in this paper challenges this preconception. It involves a woman who enters an indoor stairwell of a parking lot and slips on the dry concrete landing while approaching the stairs with her arm outstretched to grasp the railing. The current state-of-the-art of human slipping provides this victim with no remedy at law.

This paper presents a forensic and safety study that focuses on slip and fall. Slip is usually analyzed by a classical system that has no redeeming features. This protocol provides a go/no-go criterion that proclaims a walking surface safe if its interaction with a surrogate material (e.g. leather) produces an average coefficient of friction greater than 0.5. It turns out that many walkers slip on such mythical "safe" floors. The subject case adopts a modern theory of human slipping that quantitatively predicts the number of walkers who will slip on a given surface including concrete landings.

INTRODUCTION

The ambulation of pedestrians claims more lives and produces more disabling injuries than warfare. Every year global statistics indicate that Slip/Trip and Fall is the No. 1 cause of traumatic death and injury among senior citizens and No. 2 among the general population. The automobile is the only competition for these dubious distinctions. This paper presents a forensic and safety study that focuses on slip and fall. Slip is usually analyzed by a system that has no redeeming features. This classical protocol provides a go/no-go criterion that proclaims a walking surface safe if its interaction with a surrogate material (e.g. leather) produces an average coefficient of friction greater than 0.5. It turns out that many walkers continue to slip on such a mythical "safe" floor. The subject case adopts a modern theory of human slipping that is distinguished by the following characteristics:

- It embraces voluminous international studies of human gait and quantitatively accounts for pedestrian walking style, age, gender, health, speed and course.
- It reflects actual floor/footwear couples.
- It accounts for the distance walked.
- It explains why lower friction sometimes produces fewer slips.
- It addresses the lowest friction coefficient encountered, not the average.
- It incorporates the notion of traffic patterns and duty cycles on a walking surface.
- It quantitatively predicts the number of walkers who will slip on a given surface.

A Case Study

A middle-aged female executive was returning to a four story parking structure whose second floor was at ground level. This self-parking facility was serviced by four nominally identical stairwells that were well lighted with painted concrete landings that were dry and unobstructed with most of the paint worn away. It was a dry summer afternoon when the woman entered the street level stairwell wearing all leather low heeled sandals. As she approached the down staircase shown in Fig. 1, her left foot slipped while her right hand was extended to grasp the railing when it came into range. She fell feet first to the bottom of a flight of stairs.

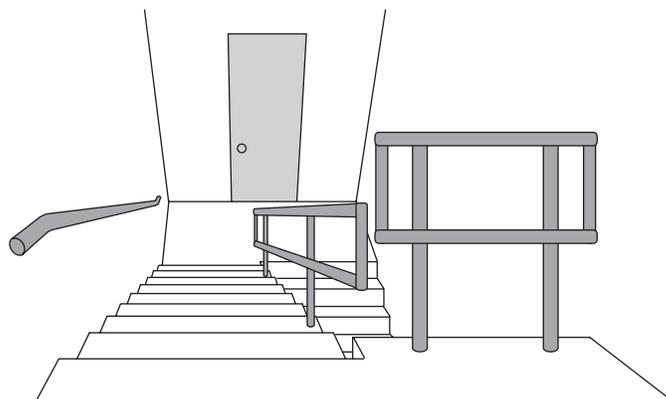


Figure 1. Case Study Stairwell and Staircase

A safety analysis of the stairwell landing began by measuring its coefficient of friction, COF, using a horizontal pull tribometer with three leather feet and following the protocol, ASTM F609.79 [1]. Table I displays the raw data associated with the 50 COF measurements and Figs. 2 and 3 display the data in a “bell shaped” probability density curve and in a cumulative distribution curve which will be used later. The average or arithmetic mean $\bar{\mu}$ of the COFs is $\bar{\mu} = 0.51$.

Table I. Fifty COF Measurements - Accident Landing

0.45	0.58	0.47	0.60	0.37
0.55	0.62	0.53	0.50	0.45
0.57	0.52	0.52	0.49	0.47
0.55	0.46	0.55	0.51	0.50
0.48	0.54	0.57	0.45	0.65
0.46	0.52	0.60	0.41	0.53
0.45	0.52	0.67	0.45	0.60
0.51	0.65	0.57	0.45	0.33
0.39	0.58	0.50	0.43	0.47
0.42	0.55	0.52	0.44	0.42
Average COF, $\mu = 0.51$				

According to conventional slip theory, a safe walking surface is defined by the inequality

$$\bar{\mu} \geq \mu_c \tag{1}$$

where μ_c is the critical coefficient of friction that is usually established by legislative fiat as opposed to rational analysis. One of the oldest and most widely recommended values for μ_c is 0.5; quite literally thousands of experts will testify that the painted concrete surface is reasonably safe because $\bar{\mu} = 0.51 > 0.5$. Furthermore, in the spirit of the *Daubert*, and other related court decisions [2-4], they can support their approach with considerable literature. The plaintiffs in similar circumstances almost never prevail in the associated product liability actions. Justice can seldom be served when conventional slip theory is embraced. A new approach has been advanced for redressing the plight of “slip and fall” victims and for mitigating the dangers of human ambulation.

According to a 1932 decision by Judge Learned Hand [5]: “Indeed, in most cases reasonable prudence is, in fact, common prudence; but strictly, it is never its measure; a *whole calling* may have unduly lagged in the adoption of new and available devices.” When applied to conventional slip analysis, this philosophy leads down two separate paths. The first discloses a group of papers that discredit conventional slip theory [6-11]. The second reveals five papers in the national and international literature that reformulate human slip theory using extreme value statistics [12-16].

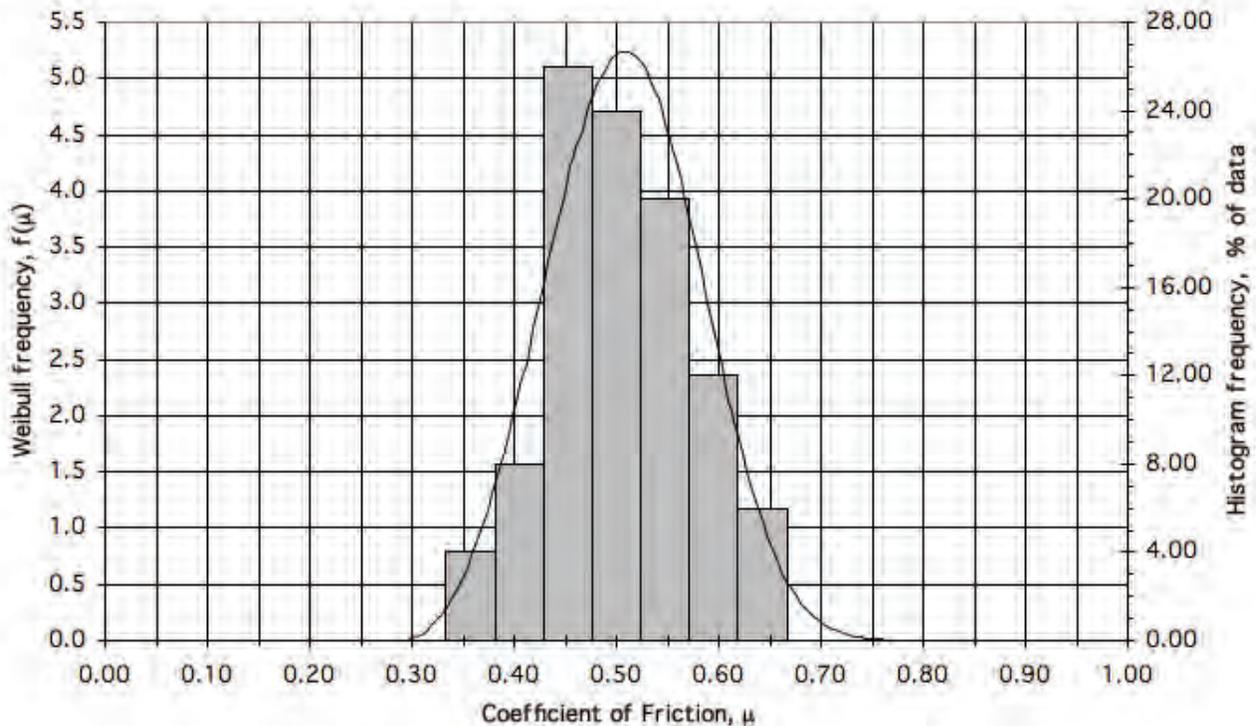


Figure 2. Probability Density Distribution Coefficients of Friction

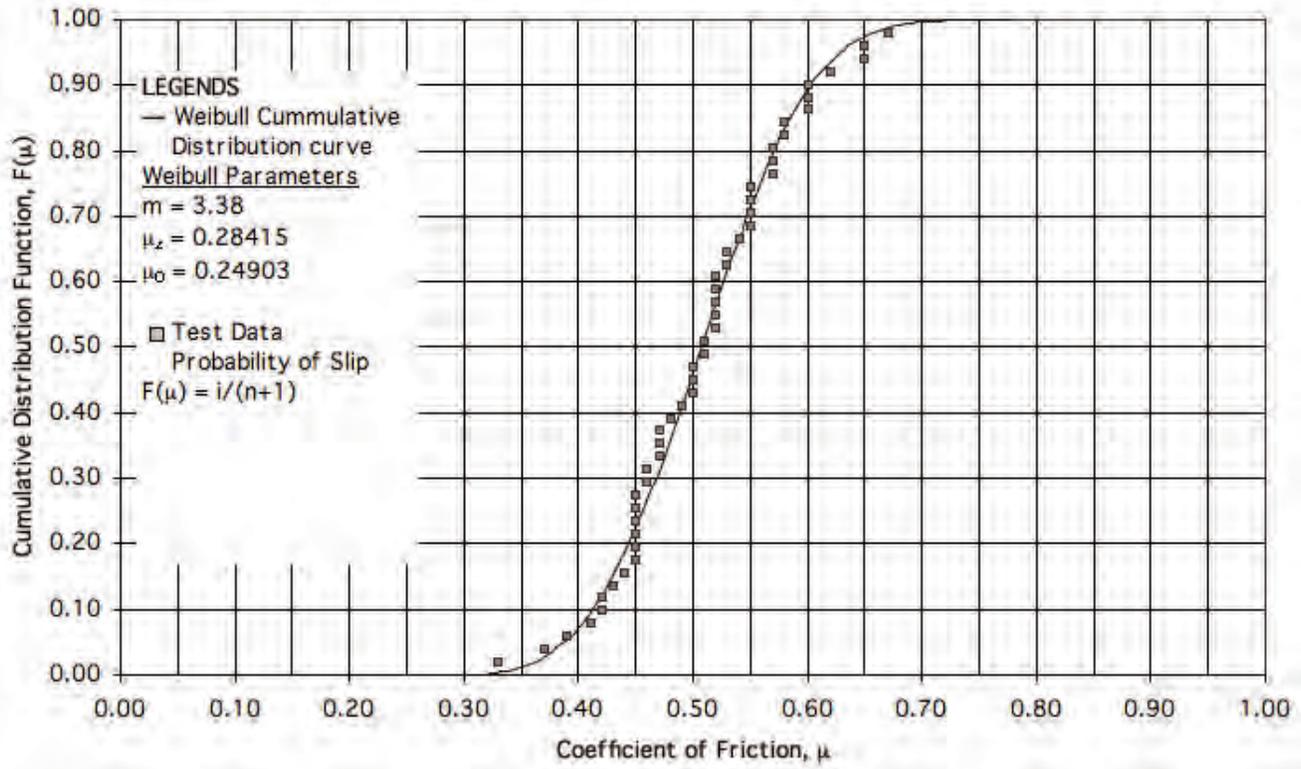


Figure 3. Cumulative Distribution Function Coefficients of Friction

APPLICATION OF REFORMULATED SLIP PROTOCOL

Preliminary Remarks

During ambulation, every maneuver causes the feet to impose a tangential loading at each contact with the floor. If the frictional resistance at the contact point is less than the associated tangential loading, slipping occurs and sometimes falling. There are five disciplines that enable one to develop the general theory for predicting the number of walkers who will slip within a given time period on a statistically homogeneous and isotropic floor with respect to friction. These include force-plate studies, floor duty cycles, tribometry, extreme value theory of slipperiness, and floor reliability theory.

Frictional Resistance - Extreme Value Statistics

If the coefficient of friction is measured throughout a walking surface, the resulting values may be presented as a “bell shaped” curve which characterizes the floor/footwear couple (see Fig. 2). To execute an n -step perambulation across the surface without slipping requires that a walker survive the step with the lowest friction. This observation has led to the development of a new theory of “slip and fall” based on extreme value statistics [17]. This theory provides that the “bell shaped” curve of friction coefficients must be of the Weibull form and that the probability

that a random friction coefficient M will not exceed μ_r , $P_r \{M \leq \mu_r\}$, is expressed by F :

$$F(\mu_r) = 1 - e^{-n \left(\frac{\mu_r - \mu_z}{\mu_0} \right)^m}, \dots \mu_r \geq \mu_z \quad (2a)$$

$$F(\mu_r) = 0, \dots \mu_r \leq \mu_z \quad (2b)$$

where μ_r is the resisting coefficient of friction for a particular floor/footwear couple; n is the number of steps taken during a given walk, and μ_0 , μ_z and m are Weibull parameters obtained from the data represented by the “bell shaped” probability density function. It should be noted that μ_z is the zero probability friction coefficient; for applied loads at or below this value, there is no risk of slipping. The probability density distribution associated with Eq. 2 is given by:

$$f(\mu_r) = \frac{nm}{\mu_0} \left(\frac{\mu_r - \mu_z}{\mu_0} \right)^{m-1} e^{-n \left(\frac{\mu_r - \mu_z}{\mu_0} \right)^m} \mu_r \geq \mu_z \quad (3a)$$

$$f(\mu_r) = 0 \quad \mu_r \leq \mu_z \quad (3b)$$

As indicated in Fig. 3, the Weibull parameters for the stairwell landing are $m = 3.38$, $\mu_z = 0.28415$ and $\mu_0 = 0.24903$. These were determined by the method of moments [18]. The number of steps n taken by the various parking lot customers is determined by the usage of the stairwell landings or duty cycle.

Duty Cycle - Landings

The four level parking facility has a capacity of 870 spaces which are evenly divided among the floors, i.e., 218 cars/floor. The popular facility is located in downtown Chicago and services office buildings and retail stores. The parking operator estimates from historical records 152% occupancy every day of the year. Without carpooling, this exposes the stairwell landings to the comings and goings of 965,352 pedestrians per year or 241,338 pedestrians from each floor per year.

The parking lot can only be accessed from ground level which is located between the first and second parking levels as shown in Fig. 4. This old facility has no elevators. The way the stairwell doors are positioned, the pathways across the landings are always curved. When entering the parking lot at street level, a walker will traverse two landings while executing 6 steps to enter the 1st or 2nd parking levels. To reach the 3rd parking level, an adult walker will encounter four landings and will exercise 14 steps. The 4th parking level requires 22 steps across six landings. In summary, the number of steps required are $n = 6$, 1st level; $n = 6$, 2nd level; $n = 14$, 3rd level and $n = 22$, 4th level.

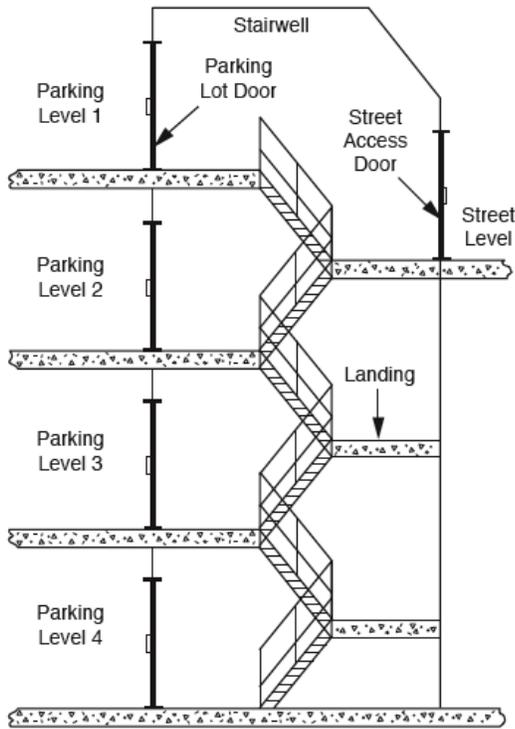


Figure 4. Typical Parking Facility Stairwell

Up to this point, the generalized slipping resistance has been addressed using tribometry, extreme value statistics and duty cycles. In the next section, we consider the generalized loading of the landings by the men and women parking their cars.

Applied Floor Loading

Gait laboratories measure the force applied to a surface by various communities of users during specific types of ambulation such as straight walking or turning. They use an instrumented walking surface called a force-plate that records the required or applied COF impressed on the surface by walking candidates. These are the applied COFs, μ_a , that must be counteracted by the resisting COFs, μ_r , to prevent slipping.

The values of μ_a are statistically distributed in a “normal bell shaped” curve with a mean $\bar{\mu}$ and a standard deviation σ . The formula for this probability density distribution $\bar{f}(\mu_a)$ is given by,

$$\bar{f}(\mu_a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2} \quad (4)$$

The applied floor friction μ_a for men and women engaged in turning ambulation is presented in Table II [19]. Recall that the pathways on the landings are curved. The demographics of the subject parking lot indicate that eighty percent of the parkers are men. This enables us to determine weighted friction parameters for μ_a ; i.e., $\bar{\mu}$ and σ . Thus,

$$\bar{\mu} = \left(\frac{0.19 + 0.22}{2}\right)(0.8) + \left(\frac{0.17 + 0.19}{2}\right)(0.2)$$

$$\bar{\mu} = 0.20$$

$$\sigma = (0.04)(0.8) + (0.02)(0.2) \quad (5)$$

$$\sigma = 0.036$$

To summarize our progress, we have established a stochastic representation of the landing’s frictional resistance $f(\mu_r)$ and a stochastic representation of the landing’s frictional loading $\bar{f}(\mu_a)$. We can now define the concept of slipping. Slip occurs whenever the applied friction force is greater than the frictional resistance, i.e.,

$$\mu_a > \mu_r \dots \text{slip criterion}$$

Reliability theory provides the tools for manipulating these two statistical worlds.

Table II. Applied Friction Loading, μ_a (after Harper, Warlow, and Clark [19])

Statistical Properties	Straight Walking		Turning			
	Men	Women	Left Foot		Right Foot	
			Men	Women	Men	Women
Mean	0.17	0.16	0.19	0.17	0.22	0.19
Standard Deviation	0.04	0.03	0.04	0.02	0.04	0.02
99.9999 Percentile	0.36	--	0.40	--	0.36	--

Floor Reliability (Slipperiness)

The reliability of a walking surface R may be defined as the probability that pedestrians will not slip during perambulation. It may be presented as the fraction of walkers who do not slip, or the percentage of walkers who do not slip, or the number of walkers who do not slip. The probability that pedestrians will slip is equal to $(1 - R)$. The floor reliability is given by [14],

$$R = \int_{-\infty}^{\infty} \bar{f}(\mu_a) \left[\int_{\mu_a}^{\infty} f(\mu_r) d\mu_r \right] d\mu_a \quad (6)$$

This well known reliability formula is discussed extensively by Kececioglu and Cormier [20]. The integrands, f and \bar{f} , are given by Eqs. 3 and 4 respectively. Unfortunately, the integration of R cannot be executed in closed form; numerical integration is required.

Floor Reliability Calculations

As shown in [14], the evaluation of R may be divided into two ranges as follows:

For $-\infty \leq \mu_a \leq \mu_z$:

$$R_1 = \int_{-\infty}^{\mu_z} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2} d\mu_a \quad (7)$$

For $\mu_z \leq \mu_a \leq \infty$:

$$R_2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu_z}^{\infty} e^{-\left[\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2 + n\left(\frac{\mu_a - \mu_z}{\mu_0}\right)^m\right]} d\mu_a \quad (8)$$

The total reliability is:

$$R = R_1 + R_2 \quad (9)$$

Calculations

Using data from Fig. 3, Eqs. 5 and the CASE STUDY, the landing reliability may be determined for the parameters displayed in Table III. Appendix A exhibits the reliability calculations performed with Eqs. 7, 8 and 9, together with the data from Table III. Table IV uses these reliabilities to determine the number of slips/year on the stairwell landings; as indicated, there will be 29.4 slips per year on the painted concrete landing. If the landings were covered with ordinary ubiquitous asphalt tiles, no slips would likely occur in a year. The reliability calculations associated with this prediction are found in Appendix A where the Weibull parameters are taken as $m = 4.75$, $\mu_z = 0.31$ and $\mu_0 = 0.40$ [17]. These values of R are used in Table V to estimate the yearly number of slips on an asphalt tile landing. It is easy to produce concrete finishes where no slipping is possible without contamination.

Table III. Parameters - Painted Concrete Landings

Applied Loading (Gaussian)	Resistance (Weibull)
$\sigma = 0.036$ $\bar{\mu} = 0.20$ $n = 6, 14, 22$	$\mu_0 = 0.24903$ $m = 3.38$ $\mu_z = 0.28415$
Level 1 + Level 2 = 482,676 pedestrians per year Level 3 = 241,338 pedestrians per year Level 4 = 241,338 pedestrians per year	

Table IV. Number of Slips In One Year - Painted Concrete Landings

Number of Steps, n	Probability of Slipping ($1-R$)	Number of Pedestrians	Number of Slips
6	15.5×10^{-6}	Level 1 + 2: 482,676	7.5
14	35.6×10^{-6}	Level 3: 241,338	8.6
22	55.0×10^{-6}	Level 4: 241,338	13.3
Total Number of Slips/Year 29.4			

Table V. Number of Slips In One Year - Asphalt Tile Landings

Number of Steps, n	Probability of Slipping ($1-R$)	Number of Pedestrians	Number of Slips
6	0.0×10^{-6}	Level 1 + 2: 482,676	0.0
14	0.0×10^{-6}	Level 3: 241,338	0.0
22	0.0×10^{-6}	Level 4: 241,338	0.0
Total Number of Slips/Year 0.0			

CONCLUSIONS AND REMARKS

- A. The painted concrete landings in the stairwells of the Chicago parking facility give rise to 29.4 pedestrian slips every year.
- B. All slips do not lead to falls. It is sometimes possible for a person to manipulate his or her body after a slip to prevent falling. On the other hand, the literature suggests that pedestrians can injure themselves while preventing the fall. Further, the railing system is often effective in preventing a fall after a slip has occurred. Unfortunately, this did not prevent the fall experienced by the woman in this case study who was reaching for the railing at the time of her injury.
- C. A properly selected landing surface should not give rise to any slipping. It is demonstrated that commonplace asphalt tile produces no slips per year under equivalent circumstances.
- D. In states that have adopted Alternative Design (Restatement of Torts, 3rd [21]), the demonstration of the behavior of asphalt tile taken together with its feasibility and modest cost are sufficient to establish liability if the landings are considered to be a product.
- E. It should be observed that the average coefficient of friction for the subject landing, 0.51, is only slightly greater than the critical friction of 0.5. This is very unusual for concrete, which normally exceeds 0.5 by a considerable margin. It should be noted that 42% of the measured friction coefficients were below 0.5.

REFERENCES

- [1] “Standard Test Method for Static Slip Resistance for Footwear, Sole, Heel, or Related Materials by Horizontal Pull Slipmeter (HPS),” 1992, ASTM F609-79 (Reapproved 1989), Annual Book of ASTM Standards, American Society for Testing and Materials, Philadelphia, pp. 313-315.
- [2] Daubert v. Merrell Dow Pharmaceuticals (92-102), 509 U.S. 579 (1993).
- [3] General Electric Co. v. Joiner (96-188), 522 U.S. 136 (1997).
- [4] Kumho Tire Co. v. Carmichael (97-1709) 526 U.S. 137 (1999).
- [5] The T.J. Hooper, 60 F.2d 737, 740 (2d Cir. 1932).
- [6] Buczek, F.L., Cavanaugh, P.R., Kulakowski, B.T., and Pradhan, P. (1990), “Slip Resistance Needs of the Mobility Disabled During Level and Grade Walking,” Slips, Stumbles, and Falls: Pedestrian Footwear and Surfaces, ASTM STP 1103, B.E. Gray, ed., American Society for Testing and Materials, Philadelphia, pp. 39-54.
- [7] Kulakowski, B.T., Buczek, F.L., Cavanaugh, P.R., and Pradhan, P. (1989), “Evaluation of Performance of Three Slip Resistance Testers,” Journal of Testing and Evaluation, 17(4), pp. 234-240.

- [8] Francis, P.R. and Zozula, C.A. (1990), "Experimental Determination of Limiting and Sliding Friction Forces for Purposes of Modeling Slips, Stumbles, and Falls," Slips, Stumbles, and Falls: Pedestrian Footwear and Surfaces, ASTM STP 1103, B.E. Gray, ed., American Society for Testing and Materials, Philadelphia, pp. 55-72.
- [9] Andres, R.O. and Chaffin, D.B. (1985), "Ergonomic Analysis of Slip-Resistance Measurement Devices," Ergonomics, **18**(7), pp. 1065-1080.
- [10] Valiant, G.A. (1987), "The Relationship Between Normal Pressure and the Friction Developed by Shoe Outsole Material on a Court Surface," Journal of Biomechanics, **20**(9), p. 892.
- [11] Schieb, D.A., Lochocki, R.F. and Gautschi, G.H. (1990), "Frictional and Ground Reaction Force Measurement with the Multicomponent Quartz Force Plate," Slips, Stumbles, and Falls: Pedestrian Footwear and Surfaces, ASTM STP 1103, B.E. Gray, ed., American Society for Testing and Materials, Philadelphia, pp. 102-112.
- [12] Barnett, R.L. (2002), "'Slip and Fall' Theory – Extreme Order Statistics," International Journal of Occupational Safety and Ergonomics (JOSE), **8**(2), pp. 135-158.
- [13] Barnett, R.L., Glowiak, S.A. and Poczynok, P.J. (2002), "Stochastic Theory of Human Slipping," Proceedings of IMECE2002, ASME International Mechanical Engineering Congress and Exposition, New Orleans, LA, pg. 1.
- [14] Barnett, R.L. and Poczynok, P.J. (2003), "Floor Reliability with Respect to 'Slip and Fall,'" Triodyne Safety Brief, **24**(3).
- [15] Barnett, R.L. and Poczynok, P.J. (2004), "Slip and Fall Characterization of Floors," Triodyne Safety Brief, **26**(2).
- [16] Barnett, R.L. and Glowiak, S.A. (2005), "Extreme Value Formulation of Human Slip – A Summary," Triodyne Safety Brief, **27**(4).
- [17] Barnett, R.L. (2002), "'Slip and Fall' Theory – Extreme Order Statistics," International Journal of Occupational Safety and Ergonomics (JOSE), **8**(2), pp.135-158.
- [18] Gregory, L.D. and Spruill, C.E. (1962), "Structural reliability of re-entry vehicles using brittle materials in the primary structure," Proceedings of the IAS Aerospace Systems Reliability Symposium, Institute of the Aerospace Sciences, New York, NY, pp. 33-55.
- [19] Harper, F.C., Warlow, W.J. and Clarke, B.L. (1961), "The forces applied to the floor by the foot in walking: I. Walking on a level surface," National Building Studies: Research Paper 32, Department of Scientific and Industrial Research, Building Research Station, London, UK.
- [20] Kececioğlu, D. and Cormier, D. (1964), "Designing a specified reliability directly into a component," Proceedings of the Third Annual Aerospace Reliability and Maintainability Conference, Society of Automotive Engineers, New York, NY, pp. 546-565.
- [21] Barnett, R.L. (1998), "Design Defect: Doctrine of Alternative Design," Triodyne Safety Brief, **13**(4).
- [22] Carnahan, B., Luther, H.A. and Wilkes, J.O. (1969), Applied Numerical Methods, New York, John Wiley and Sons, Inc.
- [23] Abramowitz, M. and Stegun, I.A., (eds), (1964), Handbook of Mathematical Functions, National Bureau of Standards Applied Mathematics Series No. 55, Washington, DC., U.S. Government Printing Office.

APPENDIX

Reliability Calculations

PAINTED CONCRETE

Reliability Calculations

using Simpson's 4-interval Rule [22]

A	B	C	D	E	F	G	H	I	J	K	L	M
1												
2												
3				Initial value $\mu_a = \mu_z =$								
4												
5												
6												
7												
8												
9												
10												
11												
12												
13												
14												
15												
16												
17												
18												
19												
20												
21												
22												
23												
24												
25												
26												
27												
28												
29												
30												
31												
32												
33												
34												
35												
36												
37												
38												
39												

INPUT	OUTPUT
μ_a	R_2
0.28	0.0000000000
0.29	0.0052487457
0.30	0.0077977675
0.31	0.0089425369
0.32	0.0094172156
0.33	0.0095986031
0.34	0.0096623459
0.35	0.0096828991
0.36	0.0096889650
0.37	0.0096905993
0.38	0.0096910002
0.39	0.0096910895
0.40	0.0096911075
0.41	0.0096911107
0.42	0.0096911113
0.43	0.0096911114
0.44	0.0096911114
0.45	0.0096911114
0.46	0.0096911114
0.47	0.0096911114
0.48	0.0096911114
0.49	0.0096911114
0.50	0.0096911114
0.51	0.0096911114
0.52	0.0096911114
0.53	0.0096911114
0.54	0.0096911114
0.55	0.0096911114
0.56	0.0096911114
0.57	0.0096911114
0.58	0.0096911114
0.59	0.0096911114
0.60	0.0096911114
0.61	0.0096911114
0.62	0.0096911114
0.63	0.0096911114

Applied Loading (Gaussian)	Symbol	Value	Units
Standard deviation	σ	0.036	
Mean value of distribution	$\bar{\mu}$	0.20	
Number of walking steps	n	6	

Resistance (Weibull)	Symbol	Value	Units
	μ_0	0.24903	
	m	3.38	
	μ_z	0.28415	

Increment of variable μ_0	$d\mu_a$
	0.01

Integration multiplier	$1/\sigma\sqrt{\pi}$
	11.08173

R_1 Calculation. $\text{Error} < 1.5 \times 10^{-7} *$
Outputs
$2.3375000000 = x = (\mu_z - \bar{\mu})/\sigma$
$0.0498673470 = d_1$ Eq. 26.2.19 *
$0.0211410061 = d_2$ Eq. 26.2.19 *
$0.0032776263 = d_3$ Eq. 26.2.19 *
$0.0000380036 = d_4$ Eq. 26.2.19 *
$0.0000488906 = d_5$ Eq. 26.2.19 *
$0.0000053830 = d_6$ Eq. 26.2.19 *
$0.9902933854 = P(x) = \Phi$ Eq. 26.2.19 *

***Handbook of Mathematical Functions [23]**

FINAL OUTPUTS	PARAMETER SYMBOL
0.9902933854	$= R_1 = \Phi$
0.0096911114	$= R_2$
0.999844967	$= R = R_1 + R_2$ Reliability
15.5E-6	$= Q = 1 - R$ Failure Probability

PAINTED CONCRETE
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M
						INPUT	OUTPUT		μ_a	OUTPUT		μ_a	OUTPUT
						μ_a	R_2			R_2			R_2
1													
2													
3						0.28	0.000000000		0.64	0.0096516338		1.00	0.0096516338
4						0.29	0.0052484552		0.65	0.0096516338		1.00	0.0096516338
5						0.30	0.0077944011		0.66	0.0096516338		1.00	0.0096516338
6						0.31	0.0089317740		0.67	0.0096516338		1.00	0.0096516338
7						0.32	0.0093970104		0.68	0.0096516338		1.00	0.0096516338
8						0.33	0.0095700705		0.69	0.0096516338		1.00	0.0096516338
9						0.34	0.0096281680		0.70	0.0096516338		1.00	0.0096516338
10						0.35	0.0096456240		0.71	0.0096516338		1.00	0.0096516338
11						0.36	0.0096502765		0.72	0.0096516338		1.00	0.0096516338
12						0.37	0.0096513662		0.73	0.0096516338		1.00	0.0096516338
13						0.38	0.0096515882		0.74	0.0096516338		1.00	0.0096516338
14						0.39	0.0096516272		0.75	0.0096516338		1.00	0.0096516338
15						0.40	0.0096516330		0.76	0.0096516338		1.00	0.0096516338
16						0.41	0.0096516338		0.77	0.0096516338		1.00	0.0096516338
17						0.42	0.0096516338		0.78	0.0096516338		1.00	0.0096516338
18						0.43	0.0096516338		0.79	0.0096516338		1.00	0.0096516338
19						0.44	0.0096516338		0.80	0.0096516338		1.00	0.0096516338
20						0.45	0.0096516338		0.81	0.0096516338		1.00	0.0096516338
21						0.46	0.0096516338		0.82	0.0096516338		1.00	0.0096516338
22						0.47	0.0096516338		0.83	0.0096516338		1.00	0.0096516338
23						0.48	0.0096516338		0.84	0.0096516338		1.00	0.0096516338
24						0.49	0.0096516338		0.85	0.0096516338		1.00	0.0096516338
25						0.50	0.0096516338		0.86	0.0096516338		1.00	0.0096516338
26						0.51	0.0096516338		0.87	0.0096516338		1.00	0.0096516338
27						0.52	0.0096516338		0.88	0.0096516338		1.00	0.0096516338
28						0.53	0.0096516338		0.89	0.0096516338		1.00	0.0096516338
29						0.54	0.0096516338		0.90	0.0096516338		1.00	0.0096516338
30						0.55	0.0096516338		0.91	0.0096516338		1.00	0.0096516338
31						0.56	0.0096516338		0.92	0.0096516338		1.00	0.0096516338
32						0.57	0.0096516338		0.93	0.0096516338		1.00	0.0096516338
33						0.58	0.0096516338		0.94	0.0096516338		1.00	0.0096516338
34						0.59	0.0096516338		0.95	0.0096516338		1.00	0.0096516338
35						0.60	0.0096516338		0.96	0.0096516338		1.00	0.0096516338
36						0.61	0.0096516338		0.97	0.0096516338		1.00	0.0096516338
37						0.62	0.0096516338		0.98	0.0096516338		1.00	0.0096516338
38						0.63	0.0096516338		0.99	0.0096516338		1.00	0.0096516338
39													

Input Parameter Name	Symbol	Value	Units
Standard deviation	σ	0.036	
Mean value of distribution	$\bar{\mu}$	0.20	
Number of walking steps	n	22	

Input Parameter Name	Symbol	Value	Units
	μ_0	0.24903	
	m	3.38	
	μ_z	0.28415	

Increment of variable μ_0	$d\mu_0$	0.01
Integration multiplier $1/\sigma\sqrt{\pi}$		11.08173
R_1 Calculation. $\text{Error} < 1.5 \times 10^{-7}$ *		
Outputs	Equations	
2.3375000000	$= x = (\mu_z - \bar{\mu})/\sigma$	
0.0498673470	$= d_1$	Eq. 26.2.19 *
0.0211410061	$= d_2$	Eq. 26.2.19 *
0.0032776263	$= d_3$	Eq. 26.2.19 *
0.0000380036	$= d_4$	Eq. 26.2.19 *
0.0000488906	$= d_5$	Eq. 26.2.19 *
0.0000053830	$= d_6$	Eq. 26.2.19 *
0.9902933854	$= P(x) = \Phi$	Eq. 26.2.19 *

***Handbook of Mathematical Functions [23]**

FINAL OUTPUTS	PARAMETER SYMBOL
0.9902933854	$= R_1 = \Phi$
0.0096516338	$= R_2$
0.9999450192	$= R = R_1 + R_2$
	Reliability
55.0E-6	$= Q = 1 - R$
	Failure Probability

ASPHALT TILES
Reliability Calculations
using Simpson's 4-interval Rule [22]

A	B	C	D	E	F	G	H	I	J	K	L	M
					INPUT	OUTPUT			OUTPUT			OUTPUT
1					μ_a	R_2		μ_a	R_2		μ_a	R_2
2												
3					0.31	0.0000000000		0.67	0.0011232116		1.00	0.0011232116
4					0.32	0.0006941595		0.68	0.0011232116		1.00	0.0011232116
5					0.33	0.0009707756		0.69	0.0011232116		1.00	0.0011232116
6					0.34	0.0010728664		0.70	0.0011232116		1.00	0.0011232116
7					0.35	0.0011077623		0.71	0.0011232116		1.00	0.0011232116
8					0.36	0.0011188088		0.72	0.0011232116		1.00	0.0011232116
9					0.37	0.0011220468		0.73	0.0011232116		1.00	0.0011232116
10					0.38	0.0011229257		0.74	0.0011232116		1.00	0.0011232116
11					0.39	0.0011231466		0.75	0.0011232116		1.00	0.0011232116
12					0.40	0.0011231979		0.76	0.0011232116		1.00	0.0011232116
13					0.41	0.0011232090		0.77	0.0011232116		1.00	0.0011232116
14					0.42	0.0011232112		0.78	0.0011232116		1.00	0.0011232116
15					0.43	0.0011232116		0.79	0.0011232116		1.00	0.0011232116
16					0.44	0.0011232116		0.80	0.0011232116		1.00	0.0011232116
17					0.45	0.0011232116		0.81	0.0011232116		1.00	0.0011232116
18					0.46	0.0011232116		0.82	0.0011232116		1.00	0.0011232116
19					0.47	0.0011232116		0.83	0.0011232116		1.00	0.0011232116
20					0.48	0.0011232116		0.84	0.0011232116		1.00	0.0011232116
21					0.49	0.0011232116		0.85	0.0011232116		1.00	0.0011232116
22					0.50	0.0011232116		0.86	0.0011232116		1.00	0.0011232116
23					0.51	0.0011232116		0.87	0.0011232116		1.00	0.0011232116
24					0.52	0.0011232116		0.88	0.0011232116		1.00	0.0011232116
25					0.53	0.0011232116		0.89	0.0011232116		1.00	0.0011232116
26					0.54	0.0011232116		0.90	0.0011232116		1.00	0.0011232116
27					0.55	0.0011232116		0.91	0.0011232116		1.00	0.0011232116
28					0.56	0.0011232116		0.92	0.0011232116		1.00	0.0011232116
29					0.57	0.0011232116		0.93	0.0011232116		1.00	0.0011232116
30					0.58	0.0011232116		0.94	0.0011232116		1.00	0.0011232116
31					0.59	0.0011232116		0.95	0.0011232116		1.00	0.0011232116
32					0.60	0.0011232116		0.96	0.0011232116		1.00	0.0011232116
33					0.61	0.0011232116		0.97	0.0011232116		1.00	0.0011232116
34					0.62	0.0011232116		0.98	0.0011232116		1.00	0.0011232116
35					0.63	0.0011232116		0.99	0.0011232116		1.00	0.0011232116
36					0.64	0.0011232116		1.00	0.0011232116		1.00	0.0011232116
37					0.65	0.0011232116		1.00	0.0011232116		1.00	0.0011232116
38					0.66	0.0011232116		1.00	0.0011232116		1.00	0.0011232116
39												

Initial value $\mu_a = \mu_z =$	Units
0.036	
0.20	
6	

Applied Loading (Gaussian)	Symbol	Value	Units
Standard deviation	σ	0.036	
Mean value of distribution	$\bar{\mu}$	0.20	
Number of walking steps	n	6	

Resistance (Weibull)	Symbol	Value	Units
	μ_0	0.40	
	m	4.75	
	μ_z	0.31	

Increment of variable μ_a	$d\mu_a$	0.01
Integration multiplier	$1/\sigma\sqrt{2\pi}$	11.08173
R₁ Calculation. Error<1.5*10⁻⁷ *		
Outputs	Equations	
3.0555555556	$= x = (\mu_z - \bar{\mu})/\sigma$	
0.0498673470	$= d_1$	Eq. 26.2.19 *
0.0211410061	$= d_2$	Eq. 26.2.19 *
0.0032776263	$= d_3$	Eq. 26.2.19 *
0.0000380036	$= d_4$	Eq. 26.2.19 *
0.0000488906	$= d_5$	Eq. 26.2.19 *
0.0000053830	$= d_6$	Eq. 26.2.19 *
0.9988768457	$= P(x) = \Phi$	Eq. 26.2.19 *

***Handbook of Mathematical Functions [23]**

FINAL OUTPUTS	PARAMETER SYMBOL
0.9988768457	$= R_1 = \Phi$
0.0011232116	$= R_2$
1.0000000574	$= R = R_1 + R_2$ Reliability
-5.74E-08	$= Q = 1 - R$ Failure Probability

ASPHALT TILES
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M
						INPUT	OUTPUT			OUTPUT			OUTPUT
						μ_a	R_2		μ_a	R_2		μ_a	R_2
1						0.31	0.0000000000		0.67	0.0011232006		1.00	0.0011232006
2						0.32	0.0006941595		0.68	0.0011232006		1.00	0.0011232006
3						0.33	0.0009707752		0.69	0.0011232006		1.00	0.0011232006
4						0.34	0.0010728645		0.70	0.0011232006		1.00	0.0011232006
5						0.35	0.0011077580		0.71	0.0011232006		1.00	0.0011232006
6						0.36	0.0011188018		0.72	0.0011232006		1.00	0.0011232006
7						0.37	0.0011220379		0.73	0.0011232006		1.00	0.0011232006
8						0.38	0.0011229156		0.74	0.0011232006		1.00	0.0011232006
9						0.39	0.0011231359		0.75	0.0011232006		1.00	0.0011232006
10						0.40	0.0011231870		0.76	0.0011232006		1.00	0.0011232006
11						0.41	0.0011231980		0.77	0.0011232006		1.00	0.0011232006
12						0.42	0.0011232001		0.78	0.0011232006		1.00	0.0011232006
13						0.43	0.0011232005		0.79	0.0011232006		1.00	0.0011232006
14						0.44	0.0011232006		0.80	0.0011232006		1.00	0.0011232006
15						0.45	0.0011232006		0.81	0.0011232006		1.00	0.0011232006
16						0.46	0.0011232006		0.82	0.0011232006		1.00	0.0011232006
17						0.47	0.0011232006		0.83	0.0011232006		1.00	0.0011232006
18						0.48	0.0011232006		0.84	0.0011232006		1.00	0.0011232006
19						0.49	0.0011232006		0.85	0.0011232006		1.00	0.0011232006
20						0.50	0.0011232006		0.86	0.0011232006		1.00	0.0011232006
21						0.51	0.0011232006		0.87	0.0011232006		1.00	0.0011232006
22						0.52	0.0011232006		0.88	0.0011232006		1.00	0.0011232006
23						0.53	0.0011232006		0.89	0.0011232006		1.00	0.0011232006
24						0.54	0.0011232006		0.90	0.0011232006		1.00	0.0011232006
25						0.55	0.0011232006		0.91	0.0011232006		1.00	0.0011232006
26						0.56	0.0011232006		0.92	0.0011232006		1.00	0.0011232006
27						0.57	0.0011232006		0.93	0.0011232006		1.00	0.0011232006
28						0.58	0.0011232006		0.94	0.0011232006		1.00	0.0011232006
29						0.59	0.0011232006		0.95	0.0011232006		1.00	0.0011232006
30						0.60	0.0011232006		0.96	0.0011232006		1.00	0.0011232006
31						0.61	0.0011232006		0.97	0.0011232006		1.00	0.0011232006
32						0.62	0.0011232006		0.98	0.0011232006		1.00	0.0011232006
33						0.63	0.0011232006		0.99	0.0011232006		1.00	0.0011232006
34						0.64	0.0011232006		1.00	0.0011232006		1.00	0.0011232006
35						0.65	0.0011232006		1.00	0.0011232006		1.00	0.0011232006
36						0.66	0.0011232006		1.00	0.0011232006		1.00	0.0011232006
37													
38													
39													

Input Parameter Name	Symbol	Value	Units
Standard deviation	σ	0.036	
Mean value of distribution	$\bar{\mu}$	0.20	
Number of walking steps	n	14	

Input Parameter Name	Symbol	Value	Units
	μ_0	0.40	
	m	4.75	
	μ_z	0.31	

Increment of variable	$d\mu_a$	0.01
Integration multiplier	$1/\sigma\sqrt{\pi}$	11.08173

R₁ Calculation. Error<1.5*10⁻⁷ *

Outputs	Equations
3.0555555556	$= x = (\mu_z - \bar{\mu})/\sigma$
0.0498673470	$= d_1$ Eq. 26.2.19 *
0.0211410061	$= d_2$ Eq. 26.2.19 *
0.0032776263	$= d_3$ Eq. 26.2.19 *
0.0000380036	$= d_4$ Eq. 26.2.19 *
0.0000488906	$= d_5$ Eq. 26.2.19 *
0.0000053850	$= d_6$ Eq. 26.2.19 *
0.9988768457	$= P(x) = \Phi$ Eq. 26.2.19 *

***Handbook of Mathematical Functions [23]**

FINAL OUTPUTS	PARAMETER SYMBOL
0.9988768457	$= R_1 = \Phi$
0.0011232006	$= R_2$
1.0000000463	$= R = R_1 + R_2$ Reliability
-4.63E-08	$= Q = 1 - R$ Failure Probability

ASPHALT TILES
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M
						INPUT	OUTPUT		μ_a	OUTPUT		μ_a	OUTPUT
						μ_a	R_2			R_2			R_2
1						0.31	0.0000000000		0.67	0.0011231896		1.00	0.0011231896
2						0.32	0.0006941595		0.68	0.0011231896		1.00	0.0011231896
3						0.33	0.0009707748		0.69	0.0011231896		1.00	0.0011231896
4						0.34	0.0010728626		0.70	0.0011231896		1.00	0.0011231896
5						0.35	0.0011077536		0.71	0.0011231896		1.00	0.0011231896
6						0.36	0.0011187949		0.72	0.0011231896		1.00	0.0011231896
7						0.37	0.0011220291		0.73	0.0011231896		1.00	0.0011231896
8						0.38	0.0011229056		0.74	0.0011231896		1.00	0.0011231896
9						0.39	0.0011231252		0.75	0.0011231896		1.00	0.0011231896
10						0.40	0.0011231761		0.76	0.0011231896		1.00	0.0011231896
11						0.41	0.0011231870		0.77	0.0011231896		1.00	0.0011231896
12						0.42	0.0011231891		0.78	0.0011231896		1.00	0.0011231896
13						0.43	0.0011231895		0.79	0.0011231896		1.00	0.0011231896
14						0.44	0.0011231896		0.80	0.0011231896		1.00	0.0011231896
15						0.45	0.0011231896		0.81	0.0011231896		1.00	0.0011231896
16						0.46	0.0011231896		0.82	0.0011231896		1.00	0.0011231896
17						0.47	0.0011231896		0.83	0.0011231896		1.00	0.0011231896
18						0.48	0.0011231896		0.84	0.0011231896		1.00	0.0011231896
19						0.49	0.0011231896		0.85	0.0011231896		1.00	0.0011231896
20						0.50	0.0011231896		0.86	0.0011231896		1.00	0.0011231896
21						0.51	0.0011231896		0.87	0.0011231896		1.00	0.0011231896
22						0.52	0.0011231896		0.88	0.0011231896		1.00	0.0011231896
23						0.53	0.0011231896		0.89	0.0011231896		1.00	0.0011231896
24						0.54	0.0011231896		0.90	0.0011231896		1.00	0.0011231896
25						0.55	0.0011231896		0.91	0.0011231896		1.00	0.0011231896
26						0.56	0.0011231896		0.92	0.0011231896		1.00	0.0011231896
27						0.57	0.0011231896		0.93	0.0011231896		1.00	0.0011231896
28						0.58	0.0011231896		0.94	0.0011231896		1.00	0.0011231896
29						0.59	0.0011231896		0.95	0.0011231896		1.00	0.0011231896
30						0.60	0.0011231896		0.96	0.0011231896		1.00	0.0011231896
31						0.61	0.0011231896		0.97	0.0011231896		1.00	0.0011231896
32						0.62	0.0011231896		0.98	0.0011231896		1.00	0.0011231896
33						0.63	0.0011231896		0.99	0.0011231896		1.00	0.0011231896
34						0.64	0.0011231896		1.00	0.0011231896		1.00	0.0011231896
35						0.65	0.0011231896		1.00	0.0011231896		1.00	0.0011231896
36						0.66	0.0011231896		1.00	0.0011231896		1.00	0.0011231896
37													
38													
39													

Input Parameter Name	Symbol	Value	Units
Standard deviation	σ	0.036	
Mean value of distribution	$\bar{\mu}$	0.20	
Number of walking steps	n	22	

Initial value $\mu_a = \mu_z =$

Input Parameter Name	Symbol	Value	Units
	μ_0	0.40	
	m	4.75	
	μ_z	0.31	

Increment of variable μ_0	$d\mu_0$	0.01
Integration multiplier	$1/\sigma\sqrt{\pi}$	11.08173

R₁ Calculation. |Error| < 1.5*10⁻⁷ *

Outputs	Equations
3.0555555556	$= x = (\mu_z - \bar{\mu})/\sigma$
0.0498673470	$= d_1$ Eq. 26.2.19 *
0.0211410061	$= d_2$ Eq. 26.2.19 *
0.0032776263	$= d_3$ Eq. 26.2.19 *
0.0000380036	$= d_4$ Eq. 26.2.19 *
0.0000488906	$= d_5$ Eq. 26.2.19 *
0.0000053830	$= d_6$ Eq. 26.2.19 *
0.9988768457	$= P(x) = \Phi$ Eq. 26.2.19 *

***Handbook of Mathematical Functions [23]**

FINAL OUTPUTS	PARAMETER SYMBOL
0.9988768457	$= R_1 = \Phi$
0.0011231896	$= R_2$
1.0000000353	$= R = R_1 + R_2$ Reliability
-3.53E-08	$= Q = 1 - R$ Failure Probability