Abstract

Should a class of ladders be prohibited for certain aerial work tasks? The ladder industry does not address questions of this type; indeed, it seldom provides sufficient information for determining the appropriateness of its products in a specific application. This paper proposes a methodology for establishing the safety of an A-Frame ladder for overhead ductwork installation. A force-plate, normally used in gait testing, was used to support HVAC workers while they installed a typical ductwork detail. The force-plate output characterized the loading environment which was then compared to the resistance profile of a special duty A-Frame ladder. The applied forces sometimes exceeded the lateral resistance of the ladder. This is consistent with our field experience involving six death cases of sheet metal workers.

Key Words: Force-Plate, A-Frame Ladder, Stability, Ductwork, Prohibited Work Tasks
I. Introduction

Despite a plethora of labels on A-Frame ladders, they provide inadequate guidelines for safe use in various ladder applications. Specifically, the modest sideways resistance associated with ladder tip-over is not characterized. Typically, A-Frame ladders display the admonition “Keep ladder close to the work; avoid pushing or pulling off to the side of ladders” [Ref. 1]. Notwithstanding this warning, almost all tasks performed on A-Frame ladders give rise to sideways loading.

The current investigation was undertaken in response to the death of an experienced sheet metal worker who was engaged in the task of installing circular duct work in the ceiling of a new construction project for a university. The duct work was 14 inches in diameter and involved straight runs of five feet. The duct was positioned just below the bottom chords of bar joists which supported corrugated metal decking. The concrete working surface was 14 feet below the ceiling.

At the time of the accident, the construction site had no wall panels so the decedent was in full view of everyone in the work area. The duct work contained some reductions and bends, but it was essentially unremarkable. Many crafts were working simultaneously at the time of the accident; all were within the purview of the general contractor. The worksite contained ladders, scaffolds, scissor lifts and other aerial lift devices. To accomplish his task, the decedent borrowed a 10-foot fiber reinforced plastic A-Frame ladder with a 375 lb. capacity from an
electrical subcontractor. This ladder was found on its side following the fatal fall. The general contractor and the HVAC subcontractor tacitly endorsed the use of an A-Frame ladder.

In this paper, the tip-over resistance of an A-Frame ladder is established by simple first order calculations and by testing. This resistance is compared to the distribution of lateral forces generated during actual installations of duct work. These installations were conducted by sheet metal workers standing on the surface of a force-plate as opposed to a ladder step which would have compromised their safety. Force-plates are generally used by gait laboratories to determine forces generated during ambulation; they measure horizontal and vertical forces and moments transmitted to the surface of the force-plate.

II. Ladder Tip-Over Resistance

A 10-foot “Special Duty” reinforced plastic A-Frame ladder with a rated capacity of 375 lb. was used in our testing program; the front and side elevations are depicted in Fig. 1 which shows a two-foot section of 1¼-in. pipe centrally located on the top cap. This pipe is used as a loading fixture. Its weight combined with the ladder is 50 pounds. Fig. 1 indicates that the seventh step is supporting a centrally located 218 lb. workman. The footprint of the ladder is depicted in Fig. 2a with solid lines representing the tip axes.
A. Rigid Body Force Analysis

The vertical forces shown in Fig. 1 are gravity forces $W_i$ which act downward in the negative $z$ direction. Regardless of their locations $(x_i, y_i)$, this collection of $n$ forces may be represented by a single force $W$ which acts downward at the so-called center of pressure $(\bar{x}, \bar{y})$ where,

$$W = \sum_{i=1}^{n} W_i$$  \hspace{1cm} \text{total force…} \hspace{1cm} \text{Eq. 1}$$

$$\bar{y} = \left(\frac{1}{W}\right) \sum_{i=1}^{n} y_i W_i$$ \hspace{1cm} \text{Eq. 2}$$

$$\bar{x} = \left(\frac{1}{W}\right) \sum_{i=1}^{n} x_i W_i$$ \hspace{1cm} \text{Eq. 3}$$

Using these definitions, the magnitude of the resisting or restoring moment $(M_r)$ that develops when rotation begins about the right side tipping axis of the ladder is $(M_r)_R$ which is simply, 

$$(M_r)_R = W \left( b - \bar{x} \right)$$ \hspace{1cm} \text{Eq. 4a}$$
Similarly, the magnitude of the resisting moments about the front, left side, and back tipping axes of the ladder are respectively:

\[ (M_r)_F = W \bar{y} \quad \text{Eq. 4b} \]

\[ (M_r)_L = W \bar{x} \quad \text{Eq. 4c} \]

\[ (M_r)_B = W(d - \bar{y}) \quad \text{Eq. 4d} \]

These resisting moments are illustrated in Fig. 2a where we observe that all the moments act inward; they attempt to keep the ladder from tipping in any direction. The resisting moments given by Eqs. 4 provide a bound on the moment resistance that is rectangular in shape; when plotted in \((x', y')\) coordinates, the rectangular resistance diagram is geometrically similar to the ladder footprint with a proportionality constant \(W\).

Three inferences are worth noting:

1. As a climber moves away from a tip-axis, the corresponding resistance increases, e.g., move to the left to improve the right tip resistance.

2. The corners of the ladder provide the most resistance to overturning, i.e., simultaneous tipping over two adjacent sides maximizes the resistance. Recall from statics that a moment applied anywhere on a rigid body
produces the same rotational effect [Ref. 2]. Figure 2b shows the two resisting moments \((M_r)_F\) and \((M_r)_R\) acting respectively on the front and right tip axes of the ladder. Their resultant, using vector addition, defines the maximum tipping resistance; thus,

\[
\text{Resultant } (M_r)_{F,R} = \sqrt{(M_r)_F^2 + (M_r)_R^2}
\]

Eq. 5

\[
\beta = \tan^{-1} \frac{(M_r)_F}{(M_r)_R} = \tan^{-1} \frac{y}{b-x}
\]

Eq. 6

The largest of the four corner resisting moments is associated with the largest of the four rectangles depicted in Fig. 2b. These four moments act in a direction perpendicular to the diagonals of the associated rectangles.

3. If the center of pressure is centrally located on the ladder, \(x = b/2\), the sideways resistance is symmetrical and independent of the step supporting the climber.

The prediction of stability resistance based on rigid body analysis provides an unconservative approximation for real ladders. Flexibility compromises their tipping resistance. Fully characterizing the behavior of real ladders is a daunting analysis problem fraught with geometric nonlinearities and mechanism behavior with joints that exhibit frictional resistance and rattle-space constraints and are influenced by wear behavior that is stochastic.
B. Ladder Tests

Lateral forces on ladders that do not slide are maximum at the point of incipient rotation about one or two tipping axes. These maximum lateral forces, $\bar{F}_{\text{max}}$, may be used directly to characterize the tipping resistance of a ladder. On the other hand, an equivalent but more general approach is to convert these maximum lateral forces into overturning moments $M_0$ about the tipping axes. At incipient tipping, the resisting moment $(M_r)$ is equal in magnitude to the maximum overturning moment $(M_0)_{\text{max}}$. Thus, the stability of a ladder in the direction of $\bar{F}_{\text{max}}$ may be characterized either by $|\bar{F}_{\text{max}}|$ or by the set of resisting moments, $[(M_r)_R, (M_r)_L, (M_r)_F, (M_r)_B]$.

The special duty ladder used in our test program was subjected to a force $F$ operating in a direction $\theta$ in a horizontal plane located 137.5 in. above the ladder base as shown in Figs. 1 and 3. The magnitude of the force in the direction $\theta$ was continuously increased and its maximum was recorded, $|F(\theta)|_{\text{max}}$. This corresponded physically with two legs being lifted from the ground while the ladder rotated about the line connecting the remaining legs. This test was repeated in directional increments of ten degrees as indicated in Fig. 3 where the important ladder parameters are superimposed. The test results are displayed in Fig. 4a where the origin of the force vectors is taken arbitrarily about the
Fig. 3 - Base Footprint - Ladder Parameters
Fig. 4 - Tip-Over Resistance Tests

a) Tipping Forces (lb)

Front of Ladder

Rigid Body Analysis

Testing (Best Fit)

b) Bounds on Tipping Forces;
Force Resistance Diagram
projection of the center of pressure. Note that the planar force $F$ produces the same moment about the tip axes regardless of its planar location.

Figure 4b shows the “best fit” locus of maximum forces as a rectangle. The rigid body bound on these forces is shown to circumscribe the “best fit” rectangle. The force-resistance diagram depicted in Fig. 4b can be converted into a moment-resistance diagram by simply multiplying all the forces by $H = 137.5$ inches. Indeed, $HF(0^\circ) = (M_r)_F$, $HF(90^\circ) = (M_r)_R$, $HF(180^\circ) = (M_r)_B$, and $HF(270^\circ) = (M_r)_L$. The force-resistance diagram is more intuitive. It is very important to observe that even on the new heavy-duty ladder the rigid body analysis significantly overestimates the stability. The sideways tipping resistance is 14.79% less than predicted; the fore and aft resistance is approximately 12.5% less. These errors will grow as the ladder joints loosen.

III. Duct Work Installation

The previous section characterized the strength (stability) of an A-Frame ladder; here we examine the loading environment encountered during the installation of duct work.
A. Simulation

Figure 5 illustrates the test set-up that was used to measure the forces generated by two experienced union sheet metal workers undertaking the installation of two five foot sections of 14 in. diameter ducts joined by a right angle elbow. The most significant feature of this set-up is the absence of a ladder; a force-plate has been substituted to provide a safe work platform for the HVAC workmen and to measure and record the load history. The roof system shown could be adjusted vertically in one-foot increments. The bar joists and decking are ubiquitous. Fasteners, hand tools, work belts, footwear and hard hats used to assemble and attach the ducting to the bar joists were furnished by the workmen.

In each phase of the installation, the candidates stood with their feet planted in one position on the force-plate. The initial ceiling height was adjusted to suit the preference of the workmen. This simulated their use of ladder steps of different height. The test fixture had the capability of varying the ceiling height during the installation; however, the candidates elected to simulate only one rung height. The force-plate was moved on the floor whenever a new “ladder position” was required by the workmen. To simulate the tool rest that a real ladder provides, a typical tool shelf was available on a stand.
B. Force-Plate

Falling from or with an A-Frame ladder is life-threatening. The work platform shown in Photograph 1 simulates the A-Frame ladder without exposing the candidates to the concomitant hazards. The four foot square platform has an elevation of 4.75 in. and contains a central flush mounted force-plate with plan dimensions of 20 x 18.25 inches.

Force-plates are normally used by gait laboratories to measure foot/floor contact forces. The following components comprise the force-plate system used in our program; they were all manufactured by Advanced Mechanical Technology, Inc. located at 176 Waltham Street, Watertown, MA:

1. Biomechanics Force Platform, Item: OR6-7-1000

2. 6-Channel MiniAmp Signal Conditioner/Amplifier, Item: MSA-6 w/PS Power Supply


The force-plate has the capability of measuring three orthogonal forces and three orthogonal moments applied to its surface.
Photograph 1: Candidate HVAC Workman
Supported by Force-Plate
The conditioned output signals from the force-plate are fed into a computer to record the load-history applied to an A-Frame ladder under complex tasks. The free-body diagram shown in Fig. 6a involves a workman generating a vertical force $P_v$, a horizontal force $P_h$ and a twisting moment $M$ while perched on an A-Frame ladder that supports the weight of his tools $W_T$ resting on the pail shelf. This task gives rise to base reactions – a shear force $P_h$; a vertical force $V$ equilibrating the ladder weight $W_L$, the climber's weight $W_C$, the weight of the tools $W_T$ and the vertical applied force $P_v$; and a base moment about the center of the ladder, $W_A$, which is comprised of the moment of the horizontal force $P_hH$, the moment of the vertical force $P_v e$ and the applied twist (moment) $M$.

Figure 6b shows the same A-Frame ladder with the top portion removed above the step used by the workman. All the forces acting on the step are indicated, i.e., the vertical forces $W_C, W_T$ and $P_v$, the horizontal force $P_h$ and the moment about the center $B$ which is $M_B = P_v e + (H - h)P_h + M$. Observe that the base reaction forces indicated in Fig. 6a are duplicated when the “step” forces in Fig. 6b are augmented by the additional vertical force $W_L$ and the additional moment component $P_hh$. All of the “step” forces shown in Fig. 6b, except the constant weight of the tools $W_T$, are measured and recorded by a force-plate supporting the same workman performing the same tasks.

The force-plate records the planar components of $\bar{P}$ and $\bar{M_0}$ which can be compared to the force-resistance diagram or the moment-resistance diagram.
$W_c$: Wt. of climber
$W_l$: Wt. of ladder
$W_t$: Wt. on tray or pallet shelf
$P_v$: Lifting by climber
$P_h$: Horizontal pull by climber
$M$: Applied torque by climber

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**Fig. 6 - Ladder Loading and Reactions**

- **a) Free-Body Diagram**
  
  $M_x = P_h H + P_v e + M$
  $V = W_l + W_c + W_t + P_v$

- **b) Equivalent Free-Body Diagram**
  
  $M_x = P_h H + P_v e + M$
  $V = W_l + (W_c + W_t + P_v)$
respectively to determine a ladder’s stability. These data may be applied to any ladder and any workman standing on any step. Force-plate data characterize the load history for a dedicated task.

C. Test Results

On separate days, each of two experienced HVAC sheet metal workers installed an L-shaped ductwork detail consisting of two five-foot sections of 14 in. diameter 26 gage sheet metal tubes and one right angle elbow. The three components each weighed less than 15 lbs. and each was lifted to the bottom chords of regularly spaced bar joists on the shoulders of the worker. A number of common elements were observed during the installations.

- Two dedicated straps were used to secure each of the straight tubes.
- A cordless drill was used with self-drilling screws to fasten the straight sections to the elbow.
- Each candidate repositioned the force-plate three times.
- Both workmen, when supported on the force-plate, stood in a single stance.
- During both installations, the force-plate was used for about fifteen minutes.
- The lateral forces developed by the two HVAC sheet metal workers were applied in a narrow horizontal corridor just above their shoulders.
• The lightweight components were supported on the worker’s shoulders until they were strapped or fastened in place.
• Vertical forces were applied infrequently without reaching.
• Each candidate chose a single ceiling height.
• The net vertical forces for each candidate varied only slightly during the installations.

During the installations, the output of the force-plate was sampled and recorded fifty times per second using the digital AMTI Mini-Amp M5A-6. The resultants of the recorded lateral forces are presented in Tables I and II for the respective candidates. Because the resistance of the test ladder was measured at an elevation of \( H = 137.5 \) in. to coincide with the shoulder height of a specific worker during a specific installation, Tables I and II can be compared directly with Fig. 4. This figure shows that the minimum sideways tipping resistance is only 26.5 lbs. Candidate I developed transverse forces greater than 26.5 lbs. during 0.321% of the work time; Candidate II exceeded 26.5 lbs. 0.095% of the time.

The task under study generates lateral forces that exceed the tipping resistance in the x-direction (sideways). This is a dangerous situation that is mitigated by a number of factors:
### Table I
**Candidate I – Lateral Force Generation**

<table>
<thead>
<tr>
<th>Lateral Force (lb.)</th>
<th>Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Mark</td>
<td>Range</td>
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<tr>
<td>2.5</td>
<td>$0 &lt; F \leq 5$</td>
</tr>
<tr>
<td>7.5</td>
<td>$5 &lt; F \leq 10$</td>
</tr>
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<td>12.5</td>
<td>$10 &lt; F \leq 15$</td>
</tr>
<tr>
<td>17.5</td>
<td>$15 &lt; F \leq 20$</td>
</tr>
<tr>
<td>22.5</td>
<td>$20 &lt; F \leq 25$</td>
</tr>
<tr>
<td>27.5</td>
<td>$25 &lt; F \leq 30$</td>
</tr>
<tr>
<td>32.5</td>
<td>$30 &lt; F \leq 35$</td>
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<td>37.5</td>
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<td>42.5</td>
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<td>47.5</td>
<td>$45 &lt; F \leq 50$</td>
</tr>
<tr>
<td>52.5</td>
<td>$50 &lt; F \leq 55$</td>
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</tbody>
</table>

### Table II
**Candidate II - Lateral Force Generation**

<table>
<thead>
<tr>
<th>Lateral Force (lb.)</th>
<th>Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Mark</td>
<td>Range</td>
</tr>
<tr>
<td>2.5</td>
<td>$0 &lt; F \leq 5$</td>
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<tr>
<td>22.5</td>
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<tr>
<td>27.5</td>
<td>$25 &lt; F \leq 30$</td>
</tr>
<tr>
<td>32.5</td>
<td>$30 &lt; F \leq 35$</td>
</tr>
</tbody>
</table>
• The critical lateral forces occur infrequently during the task.
• All ladder orientations can resist the critical lateral loading except in the x-direction.
• Support provided by the ducting itself helps to stabilize the ladder.
• Lateral forces of short duration may not tip over the ladder.
• The critical lateral forces may coincide with enhanced resistance created by compressive vertical forces.
• Heavier workmen and tools increase the ladder’s sideways resistance.

The task defined in the present study can easily be applied to other ladder configurations and types by referring to Fig. 6b where \( \mathbf{P}_v = \mathbf{M} = \mathbf{e} = 0 \). The force-plate established \( \mathbf{P}_h \) and verified \( \mathbf{W}_c \). The tipping resistance of any other ladder can be approximated by calculation or determined by testing.

IV. Concluding Remarks

A. The force-plate provides a means for characterizing a work task performed on a ladder.

B. The generalized force-time records generated by the force-plate system can be applied to any ladder and worker configuration.
C. Unsafe activities performed on a ladder can be safely undertaken while standing on a force-plate.

D. For a given task, the force-plate may exaggerate the force levels which will lead to a conservative characterization of the generalized loading. Workers may perform more vigorously from the force-plate platform which is larger, stiffer, and safer than a ladder step.

E. The output of the force-plate system can be presented as a planar force profile similar to the force-resistance diagrams in Fig. 4. This might be useful for characterizing tasks that are anisotropic.

F. Rigid body calculations provide an unconservative approximation of the tipping resistance of an A-Frame ladder. A fifteen percent error was obtained using a new extra heavy duty exemplar.

G. The **tipping resistance prediction error** associated with rigid body analysis will increase with ladder usage as the joint flexibility increases.

H. The maximum overturning resistance of an A-Frame ladder is generated by tipping over a single foot. The optimum orientation is easily determined when the center of pressure is known.
I. A-Frame ladders should always be used with fall protection when installing ceiling mounted ductwork.

J. Manufacturers should devise a warning label that communicates the impoverished nature of A-Frame sideways resistance. For example, “Sideways Resistance May Be Only X-Percent of the Climber’s Weight.”

K. Safety organizations should enlist the services of gait laboratories to develop categories of “A-Frame Prohibited Tasks.”

References


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