

SLIPPING ON CONCRETE: A CASE STUDY

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ABSTRACT

A woman enters a stairwell in a parking facility and slips on the dry concrete landing while approaching the stairs with a hand outstretched to grasp the railing. The state-of-the-art of human slipping is so impoverished that this victim can expect no remedy at law and, as a consequence, no remedial technical solution will be pursued.

This paper presents a forensic and safety study that focuses on slip and fall. Slip is usually analyzed by a system that has no redeeming features. This classical protocol provides a go/no-go criterion that proclaims a walking surface safe if its interaction with a surrogate material (e.g. leather) produces an average coefficient of friction greater than 0.5. It turns out that many walkers may continue to slip on such a mythical "safe" floor. The subject case adopts a modern theory of human slipping that quantitatively predicts the number of walkers who will slip on a given surface.

KEYWORDS: Slip and fall, floor reliability, extreme value, slips per year, dry friction

INTRODUCTION

The ambulation of pedestrians claims more lives and produces more disabling injuries than warfare. Every year global statistics on Slip/Trip and Fall indicate that senior citizens and the general population have respectively achieved No. 1 and No. 2 status with respect to traumatic death and injury. The automobile is the only competition for this dubious distinction. This paper presents a forensic and safety study that focuses on slip and fall. Slip is usually analyzed by a system that has no redeeming features. This classical protocol provides a go/no-go criterion that proclaims a walking surface safe if its interaction with a surrogate material (e.g. leather) produces an average coefficient of friction greater than 0.5. It turns out that many walkers continue to slip on such a mythical "safe" floor. The subject case adopts a modern theory of human slipping that is distinguished by the following characteristics:

- It embraces voluminous international studies of human gait and quantitatively accounts for pedestrian walking style, age, gender, health, speed and course.
- It reflects actual floor/footwear couples.
- It accounts for the distance walked.
- It explains why lower friction sometimes produces fewer slips.
- It addresses the lowest friction coefficient encountered not the average.
- It incorporates the notion of traffic patterns and duty cycles on a walking surface.
- It quantitatively predicts the number of walkers who will slip on a given surface.

A CASE STUDY

A middle-aged female executive was returning to a four story parking structure whose second floor was at ground level. This self-parking facility was serviced by four nominally identical stairwells that were well lighted with painted concrete landings that were dry and unobstructed. It was a dry summer afternoon when the woman entered the street level stairwell wearing all leather low heeled sandals. As she approached the down staircase shown in Fig. 1, her left foot slipped while her right hand was extended to grasp the railing when it came into range. She fell feet first to the bottom of a flight of stairs.

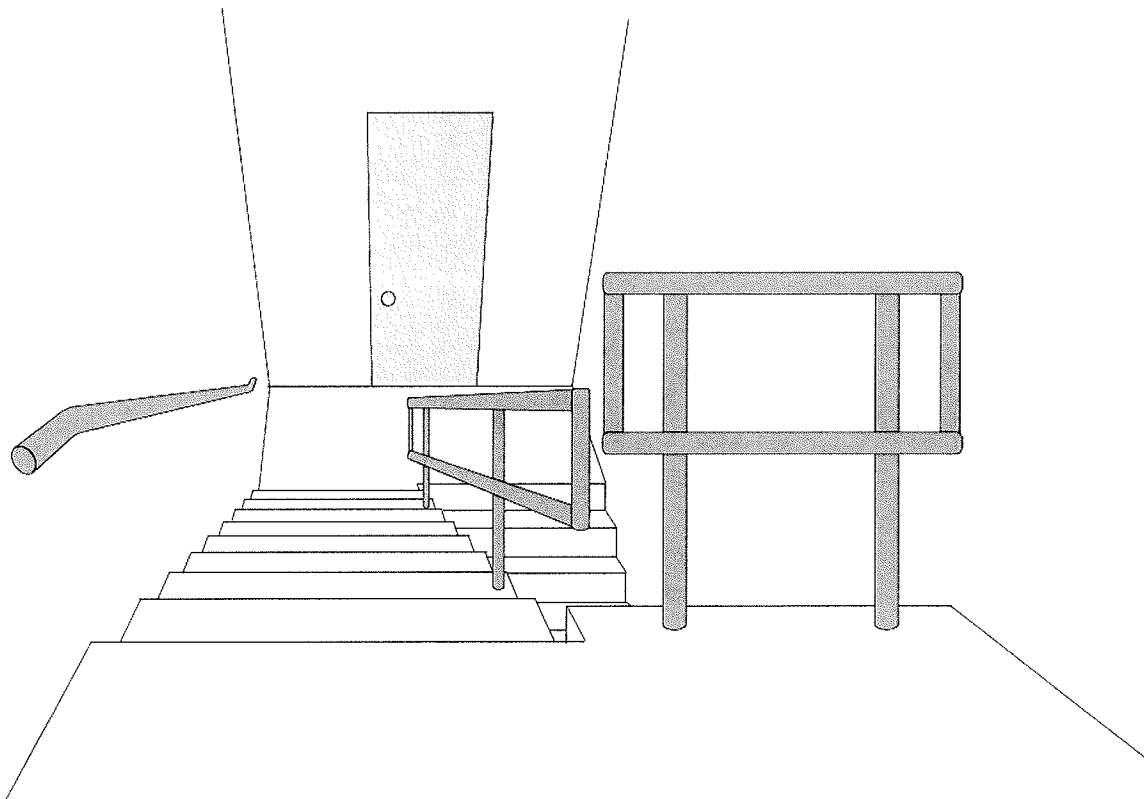


Fig. 1 Case Study Stairwell And Staircase

A safety analysis of the stairwell landing began by measuring its coefficient of friction, COF, using a horizontal pull tribometer with three leather feet and following the protocol, ASTM F609.79 [1]. Table I displays the raw data associated with the 50 COF measurements and Figs. 2 and 3 display respectively the data in a “bell shaped” probability density curve and in a cumulative distribution curve which will be used later. The average or arithmetic mean $\bar{\mu}$ of the COFs is $\bar{\mu} = 0.51$.

Table I. Fifty COF Measurements – Accident Landing

0.45	0.58	0.47	0.60	0.37
0.55	0.62	0.53	0.50	0.45
0.57	0.52	0.52	0.49	0.47
0.55	0.46	0.55	0.51	0.50
0.48	0.54	0.57	0.45	0.65
0.46	0.52	0.60	0.41	0.53
0.45	0.52	0.67	0.45	0.60
0.51	0.65	0.57	0.45	0.33
0.39	0.58	0.50	0.43	0.47
0.42	0.55	0.52	0.44	0.42
Average COF, $\mu = 0.51$				

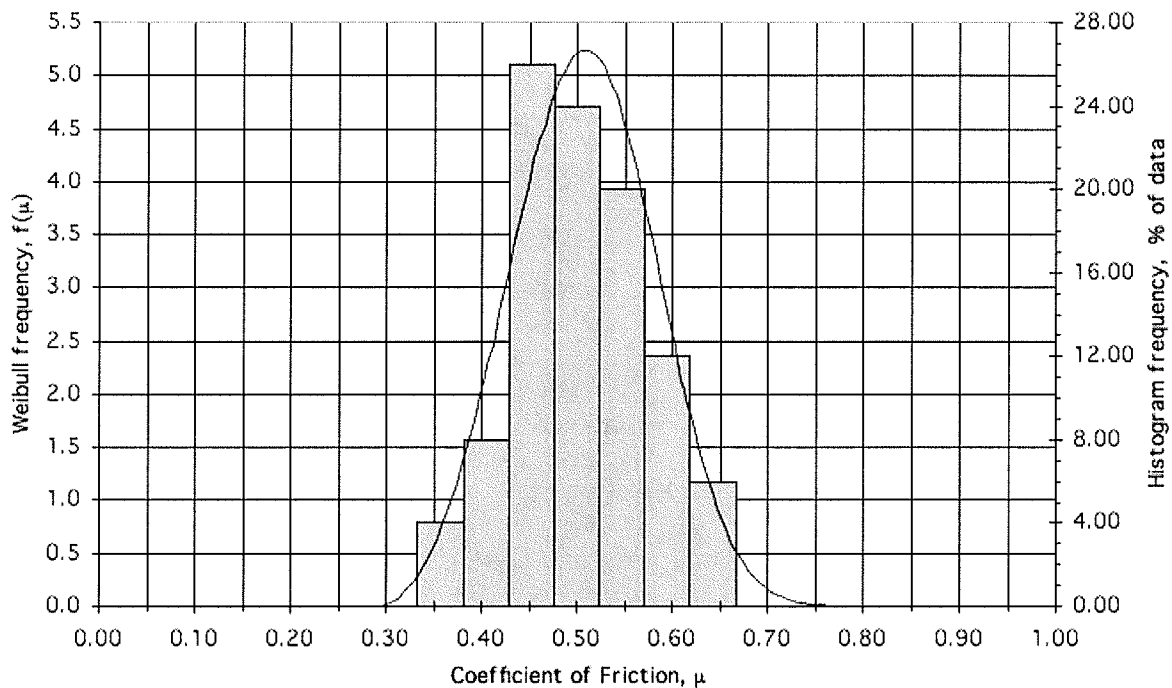


Fig. 2 Probability Density Distribution Coefficients of Friction

According to conventional slip theory, a safe walking surface is defined by the inequality

$$\bar{\mu} \geq \mu_c \quad \text{Eq. 1}$$

Where μ_c is the critical coefficient of friction that is usually established by legislative fiat as opposed to rational analysis. One of the oldest and most widely recommended values for μ_c is 0.5; quite literally thousands of experts will testify that the painted concrete surface is reasonably safe because $\bar{\mu} = 0.51 > 0.5$. Furthermore, in the spirit of the Daubert, and other related court decisions [2-4], they can support their approach with considerable literature. The plaintiffs in similar circumstances almost never prevail in the associated product liability actions. Justice cannot be served when conventional slip theory is embraced. A new approach has been advanced for redressing the plight of “slip and fall” victims and for mitigating the dangers of human ambulation.

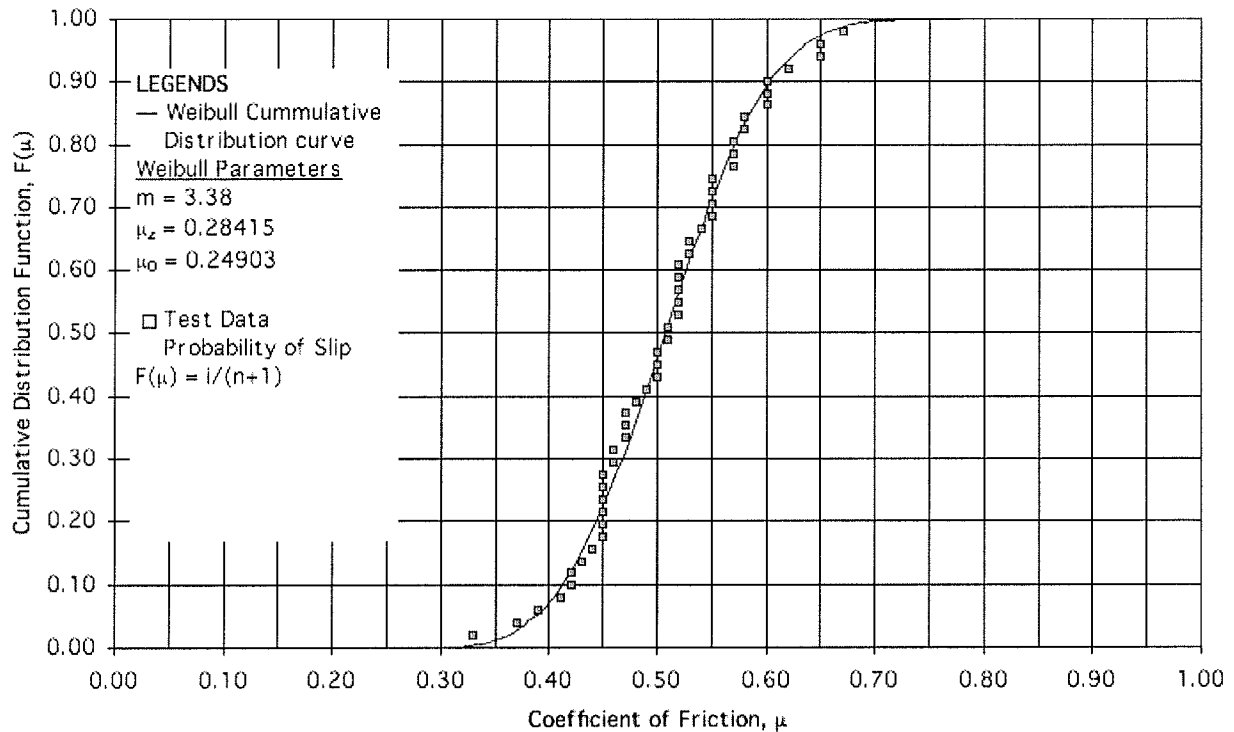


Fig. 3 Cumulative Distribution Function Coefficients of Friction

According to a 1932 decision by Judge Learned Hand [5]: “Indeed, in most cases reasonable prudence is, in fact, common prudence; but strictly, it is never its measure; a *whole calling* may have unduly lagged in the adoption of new and available devices.” When applied to conventional slip analysis, this philosophy leads down two separate paths. The first discloses a group of papers that discredit conventional slip theory [6-11]. The second reveals five papers in the national and international literature that reformulate human slip theory using extreme value statistics [12-16].

APPLICATION OF REFORMULATED SLIP PROTOCOL

Preliminary Remarks

During ambulation, every maneuver causes the feet to impose a tangential loading at each contact with the floor. If the frictional resistance at the contact point is less than the associated tangential loading, slipping occurs and sometimes falling. There are five disciplines that enable one to develop the general theory for predicting the number of walkers who will slip within a given time period on a statistically homogeneous and isotropic floor with respect to friction. These include force-plate studies, floor duty cycles, tribometry, extreme value theory of slipperiness, and floor reliability theory.

Frictional Resistance – Extreme Value Statistics

If the coefficient of friction is measured throughout a walking surface, the resulting values may be presented as a “bell shaped” curve which characterizes the floor/footwear couple (see Fig. 2). To execute an n -step perambulation across the surface without slipping requires that a walker survive the step with the lowest friction. This observation has led to the development of a new theory of “slip and fall” based on extreme value statistics, Barnett, [17]. This theory provides that the “bell shaped” curve of friction coefficients must be of the Weibull form and that the probability that a random friction coefficient M will not exceed μ_r , $P_r \{ M \leq \mu_r \}$, is expressed by F :

$$F(\mu_r) = 1 - e^{-n \left(\frac{\mu_r - \mu_z}{\mu_0} \right)^m}, \quad \dots \mu_r \geq \mu_z \quad \text{Eq. 2a}$$

$$F(\mu_r) = 0, \quad \dots \mu_r \leq \mu_z \quad \text{Eq. 2b}$$

where μ_r is the resisting coefficient of friction for a particular floor/footwear couple; n is the number of steps taken during a given walk, and μ_0 , μ_z and m are Weibull parameters obtained from the data represented by the “bell shaped” probability density function. It should be noted that μ_z is the zero probability friction coefficient; for applied loads at or below this value there is no risk of slipping. The probability density distribution associated with Eq. 2 is given by:

$$f(\mu_r) = \frac{nm}{\mu_0} \left(\frac{\mu_r - \mu_z}{\mu_0} \right)^{m-1} e^{-n \left(\frac{\mu_r - \mu_z}{\mu_0} \right)^m} \quad \mu_r \geq \mu_z \quad \text{Eq. 3a}$$

$$f(\mu_r) = 0 \quad \mu_r \leq \mu_z \quad \text{Eq. 3b}$$

As indicated in Fig. 3, the Weibull parameters for the stairwell landing are $m = 3.38$, $\mu_z = 0.28415$ and $\mu_0 = 0.24903$. These were determined by the method of moments [18]. The number of steps n taken by the various parking lot customers is determined by the usage of the stairwell landings or duty cycle.

Duty Cycle – Landings

The four level parking facility has a capacity of 870 spaces which are evenly divided among the floors, i.e., 218 cars/floor. The popular facility is located in downtown Chicago and services office buildings and retail stores. The parking operator estimates from historical records 152% occupancy every day of the year. Without carpooling, this exposes the stairwell landings to the comings and goings of 965,352 pedestrians per year or 241,338 pedestrians from each floor per year.

The parking lot can only be accessed from ground level which is located between the first and second parking levels as shown in Fig. 4. This old facility has no elevators. The way the stairwell doors are positioned the pathways across the landings are always curved. When entering the parking lot at street level, a walker will traverse two landings while executing 6 steps to enter the 1st or 2nd parking levels. To reach the 3rd parking level, an adult walker will encounter four landings and will exercise 14 steps. The 4th parking level requires 22 steps across six landings. In summary, the number of steps required are $n = 6$, 1st level; $n = 6$, 2nd level; $n = 14$, 3rd level and $n = 22$, 4th level.

Up to this point, the generalized slipping resistance has been addressed using tribometry, extreme value statistics and duty cycles. In the next section we consider the generalized loading of the landings by the men and women parking their cars.

Applied Floor Loading

Gait laboratories measure the force applied to a surface by various communities of users during specific types of ambulation such as straight walking or turning. They use an instrumented walking surface called a forceplate that records the required or applied COF impressed on the surface by walking candidates. These are the applied COFs, μ_a , that must be counteracted by the resisting COFs, μ_r , to prevent slipping.

The values of μ_a are statistically distributed in a “normal bell shaped” curve with a mean $\bar{\mu}$ and a standard deviation σ . The formula for this probability density distribution $\bar{f}(\mu_a)$ is given by,

$$\bar{f}(\mu_a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\mu_a - \bar{\mu}}{\sigma} \right)^2} \quad \text{Eq. 4}$$

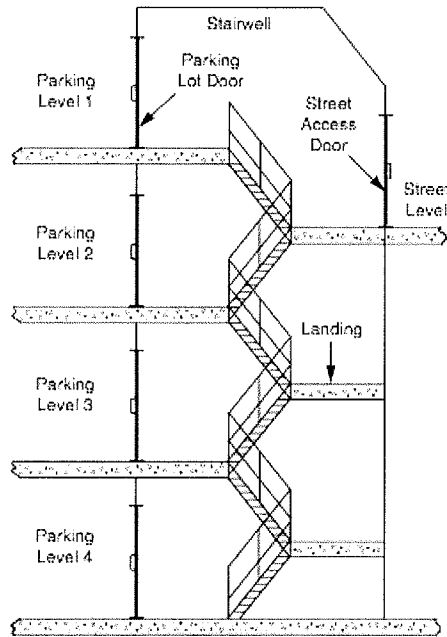


Figure 4 - Typical Parking Facility Stairwell

The applied floor friction μ_a for men and women engaged in turning ambulation is presented in Table II [19]. Recall that the pathways on the landings are curved. The demographics of the subject parking lot indicate that eighty percent of the parkers are men. This enables us to determine weighted friction parameters for μ_a ; i.e., $\bar{\mu}$ and σ . Thus,

Table II. Applied Friction Loading, μ_a (after Harper, Warlow, and Clark, [19])

Statistical Properties	Straight Walking		Turning			
	Men	Women	Left Foot		Right Foot	
			Men	Women	Men	Women
Mean	0.17	0.16	0.19	0.17	0.22	0.19
Standard Deviation	0.04	0.03	0.04	0.02	0.04	0.02
99.9999 Percentile	0.36	--	0.40	--	0.36	--

$$\bar{\mu} = \left(\frac{0.19 + 0.22}{2} \right) (0.8) + \left(\frac{0.17 + 0.19}{2} \right) (0.2)$$

$$\bar{\mu} = 0.20$$

$$\sigma = (0.04)(0.8) + (0.02)(0.2)$$

$$\sigma = 0.036$$

Eq. 5

To summarize our progress, we have established a stochastic representation of the landing's frictional resistance $f(\mu_r)$ and a stochastic representation of the landing's frictional loading $f(\mu_a)$. We can now define the concept of slipping. Slip occurs whenever the applied friction force is greater than the frictional resistance, i.e.,

$$\mu_a > \mu_r \dots \text{slip criterion}$$

Reliability theory provides the tools for manipulating these two statistical worlds.

Floor Reliability (Slipperiness)

The reliability of a walking surface R may be defined as the probability that pedestrians will not slip during perambulation. It may be presented as the fraction of walkers who do not slip, or the percentage of walkers who do not slip, or the number of walkers who do not slip. The probability that pedestrians will slip is equal to $(1 - R)$. The floor reliability is given by [14],

$$R = \int_{-\infty}^{\infty} \bar{f}(\mu_a) \left[\int_{\mu_a}^{\infty} f(\mu_r) d\mu_r \right] d\mu_a \tag{Eq. 6}$$

This well known reliability formula is discussed extensively by Kececioglu and Cormier [20]. The integrands, f and \bar{f} , are given by Eqs. 3 and 4 respectively. Unfortunately, the integration of R cannot be executed in closed form; numerical integration is required.

Floor Reliability Calculations

As shown in [14], the evaluation of R may be divided into two ranges as follows:

For $-\infty \leq \mu_a \leq \mu_z$:

$$R_1 = \int_{-\infty}^{\mu_z} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2} d\mu_a \tag{Eq. 7}$$

For $\mu_z \leq \mu_a \leq \infty$:

$$R_2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu_z}^{\infty} e^{-\left[\frac{1}{2}\left(\frac{\mu_a - \bar{\mu}}{\sigma}\right)^2 + n\left(\frac{\mu_a - \mu_z}{\mu_0}\right)^m \right]} d\mu_a \tag{Eq. 8}$$

The total reliability is:

$$R = R_1 + R_2 \tag{Eq. 9}$$

Calculations

Using data from Fig. 3, Eqs. 5 and the **CASE STUDY**, the landing reliability may be determined for the parameters displayed in Table III. Appendix A exhibits the reliability calculations performed with Eqs. 7, 8 and 9

Table III. Parameters – Painted Concrete Landings

Applied Loading (Gaussian)	Resistance (Weibull)
$\sigma = 0.036$	$\mu_0 = 0.24903$
$\bar{\mu} = 0.20$	$m = 3.38$
$n = 6, 14, 22$	$\mu_z = 0.28415$
Level 1 + Level 2 = 482,676 pedestrians per year	
Level 3 = 241,338 pedestrians per year	
Level 4 = 241,338 pedestrians per year	

together with the data from Table III. Table IV uses these reliabilities to determine the number of slips/year on the stairwell landings; as indicated there will be 29.4 slips per year on the painted concrete landing. If the landings were covered with ordinary ubiquitous asphalt tiles, no slips would likely occur in a year. The reliability calculations



associated with this prediction are found in Appendix A where the Weibull parameters are taken as $m = 4.75$, $\mu_z = 0.31$ and $\mu_0 = 0.40$ [17]. These values of R are used in Table V to estimate the yearly number of slips on an asphalt tile landing. It is easy to produce concrete finishes where no slipping is possible without contamination.

Table IV. Number of Slips In One Year – Painted Concrete Landings

Number of Steps, n	Probability of Slipping (1-R)	Number of Pedestrians	Number of Slips
6	15.5×10^{-6}	Level 1 + 2: 482,676	7.5
14	35.6×10^{-6}	Level 3: 241,338	8.6
22	55.0×10^{-6}	Level 4: 241,338	13.3
Total Number of Slips/Year			29.4

Table V. Number of Slips In One Year – Asphalt Tile Landings

Number of Steps, n	Probability of Slipping (1-R)	Number of Pedestrians	Number of Slips
6	0.0×10^{-6}	Level 1 + 2: 482,676	0.0
14	0.0×10^{-6}	Level 3: 241,338	0.0
22	0.0×10^{-6}	Level 4: 241,338	0.0
Total Number of Slips/Year			0.0

CONCLUSIONS AND REMARKS

- A. The painted concrete landings in the stairwells of the Chicago parking facility give rise to 29.4 pedestrian slips every year.
- B. All slips do not lead to falls. It is sometimes possible for a person to manipulate their body after a slip to prevent falling. On the other hand, the literature suggests that pedestrians can injure themselves while preventing the fall. Further, the railing system is often effective in preventing a fall after a slip has occurred. Unfortunately this did not prevent the fall experienced by the woman in this case study who was reaching for the railing at the time of her injury.
- C. The proper selection of a landing surface should not give rise to any slipping. It is demonstrated that commonplace asphalt tile produces no slips per year under equivalent circumstances.
- D. In states that have adopted Alternative Design (Restatement of Torts, 3rd [21]), the demonstration of the behavior of asphalt tile taken together with its feasibility and modest cost are sufficient to establish liability if the landings are considered to be a product.
- E. It should be observed that the average coefficient of friction for the subject landing, 0.51, is only slightly greater than the critical friction of 0.5. This is very unusual for concrete which normally exceeds 0.5 by a considerable margin. It should be noted that 42% of the measured friction coefficients were below 0.5.

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APPENDIX A
Reliability Calculations

PAINTED CONCRETE
Reliability Calculations
using Simpson's 4-interval Rule [22]

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<p>Initial value $\mu_a = \mu_z =$</p> <p>Applied Loading (Gaussian)</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th>Input Parameter Name</th> <th>Symbol</th> <th>Value</th> <th>Units</th> </tr> <tr> <td>Standard deviation</td> <td>σ</td> <td>0.036</td> <td></td> </tr> <tr> <td>Mean value of distribution</td> <td>$\bar{\mu}$</td> <td>0.20</td> <td></td> </tr> <tr> <td>Number of walking steps</td> <td>n</td> <td>6</td> <td></td> </tr> </table> <p>Resistance (Weibull)</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th>Input Parameter Name</th> <th>Symbol</th> <th>Value</th> <th>Units</th> </tr> <tr> <td></td> <td>μ_0</td> <td>0.24903</td> <td></td> </tr> <tr> <td></td> <td>m</td> <td>3.38</td> <td></td> </tr> <tr> <td></td> <td>μ_z</td> <td>0.28415</td> <td></td> </tr> </table> <p>Increment of variable μ_a $d\mu_a$ 0.01</p> <p>Integration multiplier $1/\sigma\sqrt{\pi}$ $1/\sigma\sqrt{\pi}(2\pi)$ 11.08173</p>	Input Parameter Name	Symbol	Value	Units	Standard deviation	σ	0.036		Mean value of distribution	$\bar{\mu}$	0.20		Number of walking steps	n	6		Input Parameter Name	Symbol	Value	Units		μ_0	0.24903			m	3.38			μ_z	0.28415		<p>R₁ Calculation. Error < 1.5*10⁻⁷ *</p> <p>Outputs Equations</p> <p>$2.3375000000 = x = (\mu_z - \bar{\mu})/\sigma$ Eq. 26.2.19 *</p> <p>$0.0498673470 = d_1$ Eq. 26.2.19 *</p> <p>$0.0211410061 = d_2$ Eq. 26.2.19 *</p> <p>$0.0032776263 = d_3$ Eq. 26.2.19 *</p> <p>$0.0000380036 = d_4$ Eq. 26.2.19 *</p> <p>$0.0000488906 = d_5$ Eq. 26.2.19 *</p> <p>$0.0000053830 = d_6$ Eq. 26.2.19 *</p> <p>$0.9902933854 = P(x) = \Phi$ Eq. 26.2.19 *</p> <p>*Handbook of Mathematical Functions [23]</p> <p>FINAL OUTPUTS PARAMETER SYMBOL</p> <p>$0.9902933854 = R_1 = \Phi$</p> <p>$0.0096911114 = R_2$</p> <p>$0.9999844967 = R = R_1 + R_2$ Reliability</p> <p>$15.5E-6 = Q = 1 - R$ Failure Probability</p>
Input Parameter Name	Symbol	Value	Units																														
Standard deviation	σ	0.036																															
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	μ_z	0.28415																															



PAINTED CONCRETE
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M			
	INPUT					OUTPUT					OUTPUT					
	μ_a					R_2					μ_a			R_2		
1	Initial value $\mu_a = \mu_z =$															
2																
3																
4																
5	Applied Loading (Gaussian)															
6	Input Parameter Name	Symbol	Value	Units												
7	Standard deviation	σ	0.036													
8	Mean value of distribution	$\bar{\mu}$	0.20													
9	Number of walking steps	n	14													
10																
11	Resistance (Weibull)															
12	Input Parameter Name	Symbol	Value	Units												
13		μ_c	0.24903													
14		m	3.38													
15		μ_z	0.28415													
16																
17	Increment of variable μ_a	$d\mu_a$	0.01													
18	Integration multiplier	$1/\sigma\sqrt{\pi}$	11.08173													
19																
20	R₁ Calculation. Error < 1.5*10⁻⁷ *															
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22	$2.3375000000 = x = (\mu_z - \bar{\mu})/\sigma$															
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26	$0.0000380036 = d_4$					Eq. 26.2.19 *										
27	$0.0000488906 = d_5$					Eq. 26.2.19 *										
28	$0.0000053830 = d_6$					Eq. 26.2.19 *										
29	$0.9902933854 = P(x) = \Phi$					Eq. 26.2.19 *										
30	*Handbook of Mathematical Functions [23]															
31																
32	FINAL OUTPUTS													PARAMETER SYMBOL		
33	$0.9902933854 = R_1 = \Phi$															
34	$0.0096710599 = R_2$															
35																
36	$0.9999644453 = R = R_1 + R_2$													Reliability		
37																
38	$35.6E-6 = Q = 1 - R$													Failure Probability		
39																



PAINTED CONCRETE
 Reliability Calculations
 using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M
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Initial value $\mu_a = \mu_z =$												
Applied Loading (Gaussian)												
Input Parameter Name	Symbol	Value	Units									
Standard deviation	σ	0.036										
Mean value of distribution	$\bar{\mu}$	0.20										
Number of walking steps	n	22										
Resistance (Weibull)												
Input Parameter Name	Symbol	Value	Units									
	μ_0	0.24903										
	m	3.38										
	μ_z	0.28415										
Increment of variable μ_a $d\mu_a$ 0.01												
Integration multiplier $1/\sqrt{\pi}(2\pi)$ 11.08173												
R₁ Calculation. Error < 1.5*10⁻⁷ *												
Outputs												
2.3375000000 = x = $(\mu_z - \bar{\mu})/\sigma$												
0.0498673470 = d ₁ Eq. 26.2.19 *												
0.0211410061 = d ₂ Eq. 26.2.19 *												
0.0032776263 = d ₃ Eq. 26.2.19 *												
0.0000380036 = d ₄ Eq. 26.2.19 *												
0.0000488906 = d ₅ Eq. 26.2.19 *												
0.0000053830 = d ₆ Eq. 26.2.19 *												
0.9902933854 = P(x) = Φ Eq. 26.2.19 *												
*Handbook of Mathematical Functions [23]												
FINAL OUTPUTS PARAMETER SYMBOL												
0.9902933854 = R ₁ = Φ												
0.0096516338 = R ₂												
0.9999450192 = R = R ₁ + R ₂ Reliability												
55.0E-6 = Q = 1 - R Failure Probability												



ASPHALT TILES
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M
	INPUT					OUTPUT					OUTPUT		
	Initial value $\mu_a = \mu_z =$					μ_a					R_2		
1						0.31	0.0000000000	0.67	0.0011232116	0.67	0.0011232116	1.00	0.0011232116
2						0.32	0.0006941595	0.68	0.0011232116	0.68	0.0011232116	1.00	0.0011232116
3						0.33	0.0009707756	0.69	0.0011232116	0.69	0.0011232116	1.00	0.0011232116
4						0.34	0.0010728664	0.70	0.0011232116	0.70	0.0011232116	1.00	0.0011232116
5						0.35	0.0011077623	0.71	0.0011232116	0.71	0.0011232116	1.00	0.0011232116
6	Applied Loading (Gaussian)												
	Input Parameter Name	Symbol	Value	Units									
	Standard deviation	σ	0.036										
	Mean value of distribution	$\bar{\mu}$	0.20										
	Number of walking steps	n	6										
7						0.36	0.0011188088	0.72	0.0011232116	0.72	0.0011232116	1.00	0.0011232116
8						0.37	0.0011220468	0.73	0.0011232116	0.73	0.0011232116	1.00	0.0011232116
9						0.38	0.0011229257	0.74	0.0011232116	0.74	0.0011232116	1.00	0.0011232116
10						0.39	0.0011231466	0.75	0.0011232116	0.75	0.0011232116	1.00	0.0011232116
11						0.40	0.0011231979	0.76	0.0011232116	0.76	0.0011232116	1.00	0.0011232116
12	Resistance (Weibull)												
	Input Parameter Name	Symbol	Value	Units									
		μ_0	0.40										
		m	4.75										
		μ_z	0.31										
13						0.41	0.0011232090	0.77	0.0011232116	0.77	0.0011232116	1.00	0.0011232116
14						0.42	0.0011232112	0.78	0.0011232116	0.78	0.0011232116	1.00	0.0011232116
15						0.43	0.0011232116	0.79	0.0011232116	0.79	0.0011232116	1.00	0.0011232116
16						0.44	0.0011232116	0.80	0.0011232116	0.80	0.0011232116	1.00	0.0011232116
17						0.45	0.0011232116	0.81	0.0011232116	0.81	0.0011232116	1.00	0.0011232116
18	Increment of variable μ_a					0.01							
19	Integration multiplier $1/\sigma\sqrt{\pi}$					11.08173							
20	R₁ Calculation. Error < 1.5*10⁻⁷ *												
21	Outputs												
22	Equations												
23	$3.0555555556 = x = (\mu_z - \bar{\mu})/\sigma$												
24	Eq. 26.2.19 *												
25	Eq. 26.2.19 *												
26	Eq. 26.2.19 *												
27	Eq. 26.2.19 *												
28	Eq. 26.2.19 *												
29	Eq. 26.2.19 *												
30	Eq. 26.2.19 *												
31	Eq. 26.2.19 *												
32	*Handbook of Mathematical Functions [23]												
33	FINAL OUTPUTS												
34	PARAMETER SYMBOL												
35	0.9988768457 = R ₁ = Φ												
36	0.0011232116 = R ₂												
37	1.0000000574 = R = R ₁ + R ₂ Reliability												
38	-5.74E-08 = Q = 1 - R Failure Probability												
39													

ASPHALT TILES
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M
					INPUT	OUTPUT				OUTPUT			
					μ_a	R_2				R_2			
					μ_a	R_2				R_2			
1					0.31	0.0000000000				0.0011232006			
2					0.32	0.0006941595				0.0011232006			
3					0.33	0.0009707752				0.0011232006			
4					0.34	0.0010728645				0.0011232006			
5					0.35	0.0011077580				0.0011232006			
6					0.36	0.0011188018				0.0011232006			
7					0.37	0.0011220379				0.0011232006			
8					0.38	0.0011229156				0.0011232006			
9					0.39	0.0011231359				0.0011232006			
10					0.40	0.0011231870				0.0011232006			
11					0.41	0.0011231980				0.0011232006			
12					0.42	0.0011232001				0.0011232006			
13					0.43	0.0011232005				0.0011232006			
14					0.44	0.0011232006				0.0011232006			
15					0.45	0.0011232006				0.0011232006			
16					0.46	0.0011232006				0.0011232006			
17					0.47	0.0011232006				0.0011232006			
18					0.48	0.0011232006				0.0011232006			
19					0.49	0.0011232006				0.0011232006			
20					0.50	0.0011232006				0.0011232006			
21					0.51	0.0011232006				0.0011232006			
22					0.52	0.0011232006				0.0011232006			
23					0.53	0.0011232006				0.0011232006			
24					0.54	0.0011232006				0.0011232006			
25					0.55	0.0011232006				0.0011232006			
26					0.56	0.0011232006				0.0011232006			
27					0.57	0.0011232006				0.0011232006			
28					0.58	0.0011232006				0.0011232006			
29					0.59	0.0011232006				0.0011232006			
30					0.60	0.0011232006				0.0011232006			
31					0.61	0.0011232006				0.0011232006			
32					0.62	0.0011232006				0.0011232006			
33					0.63	0.0011232006				0.0011232006			
34					0.64	0.0011232006				0.0011232006			
35					0.65	0.0011232006				0.0011232006			
36					0.66	0.0011232006				0.0011232006			
37					0.66	0.0011232006				0.0011232006			
38					0.66	0.0011232006				0.0011232006			
39					0.66	0.0011232006				0.0011232006			

Initial value $\mu_a = \mu_z =$	
Applied Loading (Gaussian)	
Input Parameter Name	Symbol Value Units
Standard deviation	σ 0.036
Mean value of distribution	$\bar{\mu}$ 0.20
Number of walking steps	n 14

Resistance (Weibull)	
Input Parameter Name	Symbol Value Units
	μ_0 0.40
	m 4.75
	μ_z 0.31

Increment of variable μ_a	$d\mu_a$ 0.01
Integration multiplier $1/\sigma\sqrt{2\pi}$	11.08173

R₁ Calculation. Error < 1.5*10⁻⁷ *	
Outputs	Equations
3.0555555556	$x = (\mu_z - \bar{\mu})/\sigma$
0.0498673470	d_1 Eq. 26.2.19 *
0.0211410061	d_2 Eq. 26.2.19 *
0.0032776263	d_3 Eq. 26.2.19 *
0.0000380036	d_4 Eq. 26.2.19 *
0.0000488906	d_5 Eq. 26.2.19 *
0.0000053830	d_6 Eq. 26.2.19 *
0.9988768457	$P(x) = \Phi$ Eq. 26.2.19 *

***Handbook of Mathematical Functions [23]**

FINAL OUTPUTS	PARAMETER SYMBOL
0.9988768457	$R_1 = \Phi$
0.0011232006	R_2
1.0000000463	$R = R_1 + R_2$ Reliability
-4.63E-08	$Q = 1 - R$ Failure Probability

ASPHALT TILES
Reliability Calculations
using Simpson's 4-interval Rule [22]

	A	B	C	D	E	F	G	H	I	J	K	L	M	
					INPUT				OUTPUT				OUTPUT	
	Initial value $\mu_a = \mu_z =$				μ_a	R_2	μ_a	R_2	μ_a	R_2	μ_a	R_2	μ_a	R_2
1					0.31	0.0000000000	0.67	0.0011231896	1.00	0.0011231896	1.00	0.0011231896	1.00	0.0011231896
2					0.32	0.0006941595	0.68	0.0011231896	1.00	0.0011231896	1.00	0.0011231896	1.00	0.0011231896
3					0.33	0.0009707748	0.69	0.0011231896	1.00	0.0011231896	1.00	0.0011231896	1.00	0.0011231896
4					0.34	0.0010728626	0.70	0.0011231896	1.00	0.0011231896	1.00	0.0011231896	1.00	0.0011231896
5					0.35	0.0011077536	0.71	0.0011231896	1.00	0.0011231896	1.00	0.0011231896	1.00	0.0011231896
6	Applied Loading (Gaussian)													
7	Input Parameter Name	Symbol	Value	Units										
8	Standard deviation	σ	0.036											
9	Mean value of distribution	$\bar{\mu}$	0.20											
10	Number of walking steps	n	22											
11	Resistance (Weibull)													
12	Input Parameter Name	Symbol	Value	Units										
13	μ_0		0.40											
14	m		4.75											
15	μ_z		0.31											
16														
17	Increment of variable μ_0	$d\mu_0$	0.01											
18	Integration multiplier	$1/\sigma\sqrt{\pi}$	11.08173											
19														
20	R₁ Calculation. Error < 1.5*10⁻⁷ *													
21	Outputs													
22	Equations													
23	$3.0555555556 = x = (\mu_z - \bar{\mu})/\sigma$													
24	$0.0498673470 = d_1$													
25	$0.0211410061 = d_2$													
26	$0.0032776263 = d_3$													
27	$0.0000380036 = d_4$													
28	$0.0000488906 = d_5$													
29	$0.0000053830 = d_6$													
30	$0.9988768457 = P(x) = \Phi$													
31	*Handbook of Mathematical Functions [23]													
32	FINAL OUTPUTS													
33	PARAMETER SYMBOL													
34	$0.9988768457 = R_1 = \Phi$													
35	$0.0011231896 = R_2$													
36	$1.0000000353 = R = R_1 + R_2$													
37	Reliability													
38	$-3.53E-08 = Q = 1 - R$													
39	Failure Probability													